Chapter 1

INTRODUCTION

Materials and their properties

Engineering materials include both *metals* and *non-metals* which are commonly used for *Engineering purposes.* The commonly used *metals* are *Steel, Aluminum, Copper, Cast Iron etc.,* and their *alloys*. Some of the *non-metals* include *Plastic, Rubber.*

The *mechanical properties* considered while selecting and *Engineering material* for *machine component* are *strength, stiffness, ductility, harness, wear* and *corrosion resistance, shock* and *impact resistance, toughness* etc. The selection of material for a machine or a structural member is one of the most important decisions the *Designer* has to make and this is to be made before the *dimensions* of the component are determined. For this purpose, the *Designer* is to have complete knowledge of the *properties of Materials* before hand. After choosing the *material* and the related *process,* the *Designer* can proportion the member so that the internal stresses produced are well within the values associated with failure of material.

There are many parts which have no loads on them but are merely used to fill up the space. Parts may have to be designed for *corrosion resistance* they are in an environment where they are subjected to *corrosion.* Sometimes *temperature effects* are more important in design than the *stress* and *strains*. Hence it can be summed up that a *complete knowledge of materials* and their *properties* are very essential for the *Designer.*

Design Considerations

Sometimes the *strength* of an element is an *important factor* in the determination of the *Geometry and Dimensions* of the element. Here we say that *strength* is an important *Design consideration.* The term *"Design consideration"* refers to a characteristic which influences the design of element.

A number of other *Design considerations* such as *wear* and *corrosion* consideration, *thermal consideration* etc are equally important.

The following are the important *Design considerations* the *Designer* must be aware

(a) Strength (b) Reliability (c) Cost (d) Safety (e) Weight (f) Processing (g) Utility (h) Noise (i) Style (j) Shape (k) Size (l) Stiffness (m) Lubrication (n) Maintenance.

Mechanical Engineering Design

Mechanical design means *"the design of things and systems of mechanical nature* *such as machines, structures, instruments, etc."* In most of the cases, Mechanical design utilizes *mathematics, the Materials science* and *Engineering mechanics.*

"Mechanical Engineering Design" includes all Mechanical design and it includes all disciplines of Mechanical engineering such as *Thermal science, Fluid science, etc.*

The Phases of Design

The complete *design process* from *start* to *completion* is shown in figure-1.1

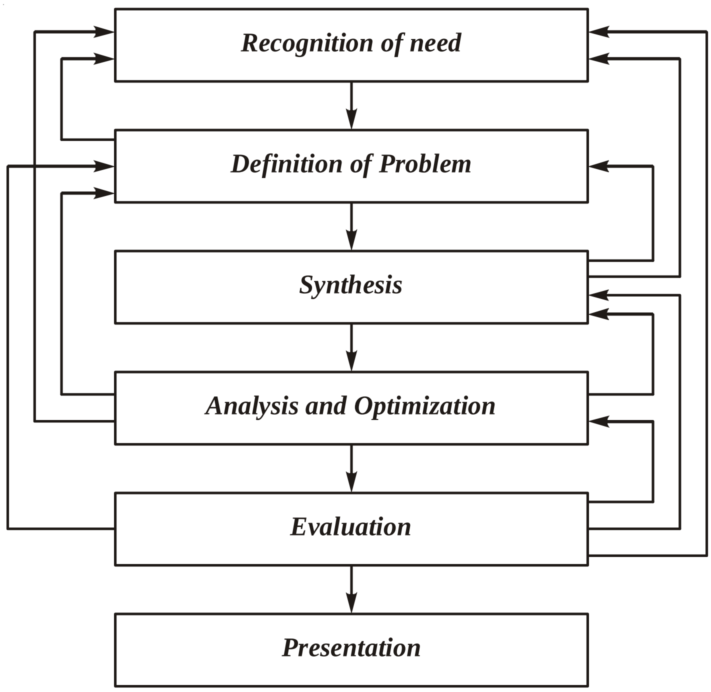

Figure-1.1: Phases of Design

The *"Design process"* begins with the *recognition* of a *need*. After many *iterations,* the *process ends* with the *presentation of plans for satisfying* the need.

(a) Recognition and Identification

Recognition of the need and *phrasing* the need constitutes a *highly creative act.* A sensitive person who is easily disturbed by things is more likely to recognize a need. It is easily recognized after one has started it.

(b) Definition of Problem

It includes the *specifications* for the things to be *designed. Specifications* define the *cost, the number to be manufactured, the expected life, the operating temperature and the reliability.*

(c) Synthesis

After the problem has been defined and a set of specifications are obtained, the next step in design is *"Synthesis"*.

(d) Analysis and Optimization

Synthesis cannot take place without both *analysis* and *optimization,* because the system under *design* must be *analyzed* to determine whether the *performance* complies with the *specifications.* The *analysis* may reveal that the system is not an *optimum one.* If the design fails either or both of these tests, the *synthesis* procedure must begin again.

(e) Evaluation and Presentation

It is significant phase of *total design process. Evaluation* is the *final proof* of *successful design* and usually involves the testing of *prototype* in the laboratory. *Communicating* the *design* to others is the important step in the *design process. Presentation* is the *final step in design process* and is *selling job.* The person who is presenting a *new solution* is attempting to *sell or prove* that the *solution* he is proposing is *better than* the *existing one.*

There are *three* means of communication; namely the *oral, written* and *graphical forms.*

A *successful Engineer* will be *technically competent* if he is versatile in all the *three forms of communication.*

Codes and Standards

In olden days, there were no standards for machine elements such as *nuts, bolts* and *screw threads.* Suppose a manufacturer produced 30mm bolts with 6mm pitch, another produced same 30mm bolts with 8mm pitch and so on. There were instances wherein an automobile mechanic in a machine shop laid out the fasteners in a line as they were dismantled for repairs so as to avoid mixing of them during reassembly. It was rather a hectic task for the *mechanics* to find replacement for a worn out or spoilt component. *There were also instances where the parts were fastened using steel wire.*

"A standard is a set of specifications for parts, materials or processes to achieve uniformity, efficiency and specified quality".

The most important *purposes of standard* is to place a *limit* on the *number* of items in and the *specifications* so as to provide reasonable *inventory* of *tooling, sizes* and *shapes.*

A *code* is a *set of specifications* for the *analysis, design, manufacture* and *construction* of something.

All the *societies* and *organizations* listed below have established specifications for *standard* and *safety* or *design codes.* Some of the *organizations* which are important for *Mechani-*

cal Engineers are

- *i)* *International Standards Organization (ISO).*
- *ii)* *British Standards Institution (BSI).*
- *iii)* *Bureau of Indian Standards (BIS).*
- *iv)* *American Society of Mechanical Engineers (ASME).*
- *v)* *American Gear Manufacturers' Association (AGMA).*
- *vi)* *Indian Institute of Foundry Men (IIF).*

Stress – Strain Diagram

The relation between the *load* on the specimen and the corresponding *deformation* obtained by conducting a *tensile / compression* test using a *Universal testing machine* are plotted with stress on the *y-axis (ordinate)* and the corresponding values of *strain* in the *x-axis (Abscissa)* as shown in figure-1.2

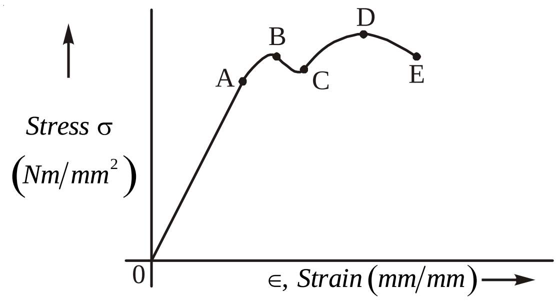

Figure-1.2: Stress-Strain Diagram

When the *load* is gradually increased, upto *point A* the strain $\in$ is directly proportional to *stress* σ *(Hooke's law)* and when the *load* on the specimen is removed, the body regains its *original shape* and *size.* The property of material to resume its *original shape* and *size* is called *elasticity.* The *stress* corresponding to the point is called *"elastic limit".* The *maximum stress* upto which the *strain* remains proportional to *stress* is called *proportional limit.* For most of the *Engineering materials, proportional limit* is very close to *elastic limit* and are often considered equal and termed *proportional elastic limit.*

An increase in *stress* above point *A* is accompanied by a *faster increase of deformation* till point *B* is reached. At point *B*, the resistance of *molecules* begins to breakdown rapidly and a *sudden* and *large increase* of *deformation* occurs without an *increase* in *load.* The *stress* corresponding to this part is called *yield point.*

After the removal of a *load* causing a *stress* greater than *elastic limit,* the body does not return to its *original shape.* The *maximum stress* at which the specimen *fails* is called the *"Ultimate Strength"* indicated by point *D.* Point *E* indicates *fracture of specimen.*

For the purpose of calculation of *stresses* and to draw the *stress-strain diagram,* the stress is based on the *original area of cross section* of member.

True Stress - Strain Diagram

True stress-strain diagrams are drawn using *instantaneous area of cross section of* specimen instead of *original area of cross section.*

Stresses due to normal and Shear Load

Many a time, the *machine elements* are subjected to the combination of *axial, bending* and *torsional* loads instead of *single load* such as *axial* or *torsional loads.* In such case, the member is subjected to *combined loads.* The *stresses* due to the *combined loads* is the *sum* of the *stresses due to the individual loading.*

Uni-axial, Bi-axial and Tri-axial state of Stress

When the member is subjected to stress due to *load* acting in *only one direction,* we say that the *stress* in the member is *Uni-axial or one dimensional stress.*

If the member is subjected to *loads* resulting in *stresses* in *two directions,* we say that the state of *stress* as *Bi-axial or two dimensional stress.*

Similarly if there are *stresses* at a point in a member in *three directions,* we say that the *state of stress* in the member as *Tri-axial or three dimensional stress.*

Sign Convention used

a) Normal Stress

Tensile stress are considered *positive* and *compressive stresses negative.* In figure-1.3, both σ_x and σ_y are positive.

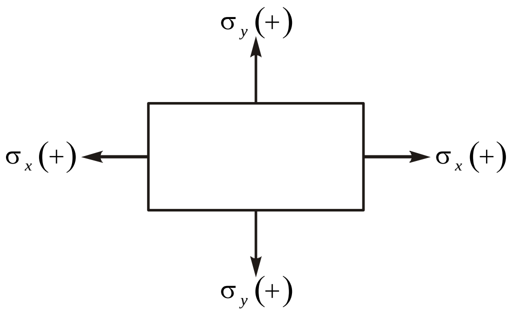

Figure-1.3

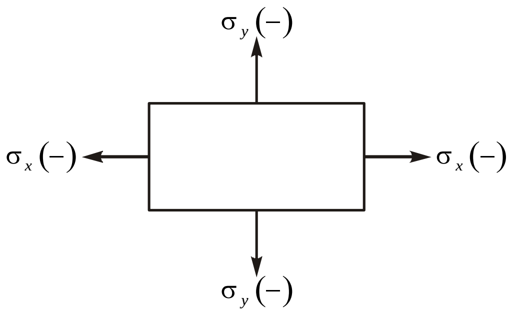

Figure-1.4

In figure-1.4, both *normal stresses* σ_x and σ_y are *negative. Normal stress* σ_x is *negative* and σ_y is *positive* in figure-1.5.

In figure-1.6, the *stress* in *x direction* is *positive* where as in the *y direction* it is *nega-*

tive.

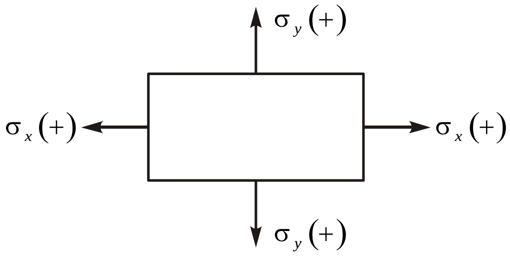

Figure-1.3

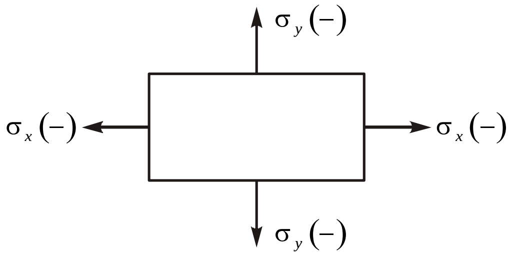

Figure-1.4

Sign Conventions for Shear Stress

The *shear stress* are considered *positive* when a *pair of shear stresses* acting on two vertical *parallel sides* of the element produce an *anti clockwise couple.* Refer figure-1.7.

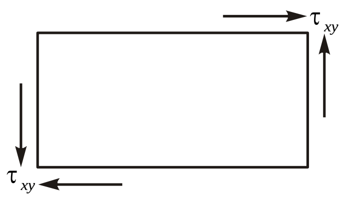

Figure-1.7: Positive Shear Stress

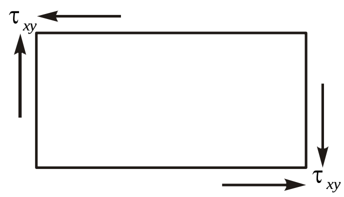

Figure-1.8: Negative Shear Stress

The *shear stress* are considered *negative* when a *pair of shear stresses* acting on two *vertical parallel sides* produce *clockwise couple* as shown in figure-1.8.

Orientation

The *angle of inclination* θ of an *inclined plane AB* with respect to vertical *AC* is considered positive when measured from *AC* in the *counter clockwise direction.*

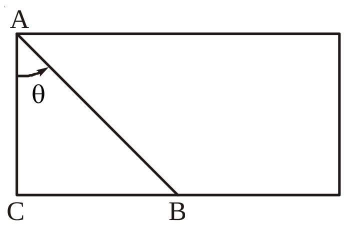

Figure-1.9

Member subjected to Uni-axial Stress

Consider a member subjected to an *axial load* resulting in *tensile stress* σ_x as shown in figure-1.10.

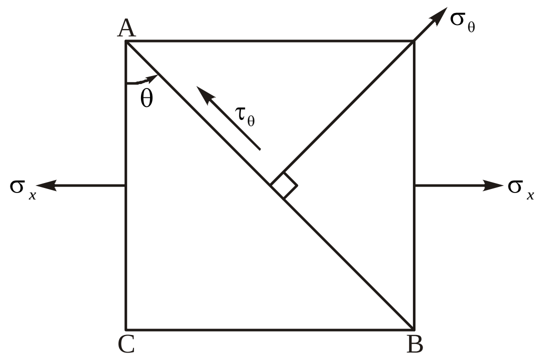

Figure-1.10

The *normal stress* σ_θ on a plane *AB* inclined at θ^0 with vertical AC is given by

$$\sigma_\theta = \sigma_x \cos^2 \theta \qquad \text{... (1)}$$

Similarly the shear stress τ_θ acting *tangential* to *AB* is given by

$$\tau_\theta = \frac{\sigma_x}{2} \sin 2\theta \qquad \text{... (2)}$$

From equation (2), the *shear stress* is *maximum* when $\sin 2\theta$ is *maximum.*

i.e., $2\theta = 90^0$ or $\theta = 45^0$.

That is the *maximum value* of *shear stress* occurs on a plane *AB* inclined at 45^0 with *AC*.

Example 1.1: A rectangular bar of section $50mm \times 25mm$ is subjected to a tensile load of 25kN. Determine the values of normal and shear stresses on a plane inclined at 30^0 with the vertical. Also calculate the magnitude and direction of the maximum shear stress. (Dec 08 – Jan 09 VTU)

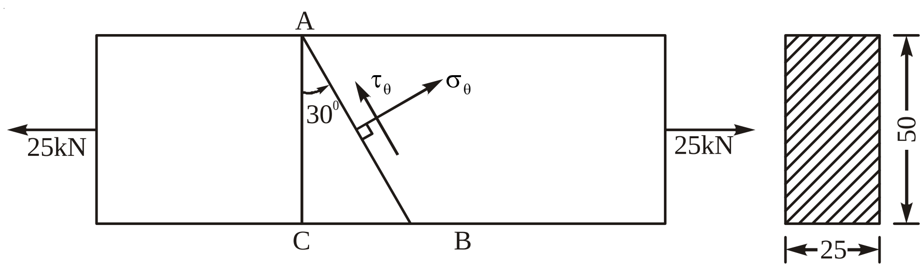

Figure-1.11

Solution:

Referring figure-1.11,

Tensile load F on member $= 25kN = 25 \times 10^3 N.$

Area of cross section A of member $= 50 \times 25 = 1250mm^2$

Therefore, *Tensile stress* $\sigma_x = \frac{F}{A} = \frac{25 \times 10^3}{1250} = 20\, N/mm^2$

i) ***Normal stress*** σ_θ ***acting normal to inclined plane AB is given by***

$$\sigma_\theta = \sigma_x \cos^2\theta = 20\cos^2 30$$

$$= 1.5\, N/mm^2$$

ii) ***Shear stress*** τ_θ ***on AB is given by***

$$\tau_\theta = \frac{\sigma_x}{2}\sin 2\theta$$

$$= \frac{20}{2}\sin 60 = 8.66\, N/mm^2$$

iii) ***Maximum shear stress*** $\tau_{\max}$ ***is given by***

$$\tau_{\max} = \frac{\sigma_x}{2} = \frac{20}{2} = 10\, N/mm^2$$

Note: $\tau_{\max}$ ***occurs on a plane inclined at*** 45^0 ***with AC.***

Example 1.2: A square beam of cross section $40mm \times 40mm$ ***is subjected to an axial pull of 16kN. Determine the values of the normal and tangential stresses on a plane inclined at*** 60^0 ***with axis of member.***

Solution:

The tensile load F on member $= 16kN = 16 \times 10^3 N$. Refer figure-1.12.

The area of cross section A of member $= 40 \times 40 = 1600mm^2$

Therefore, Tensile stress $\sigma_x = \frac{F}{A} = \frac{15 \times 10^3}{1600} = 10\, N/mm^2$

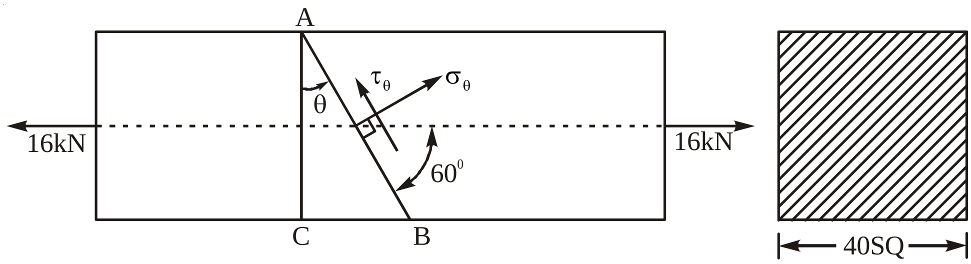

Figure-1.12

θ, the inclination of plane AB with vertical AC is $(90-60)=30^0$

Therefore, normal stress on AB $=\sigma_\theta=\sigma_x\cos^2\theta$

$$=10\cos^2 30=7.5\,N/mm^2$$

Shear stress τ_θ on AB $=\dfrac{\sigma_x}{2}\sin 2\theta$

$$=\frac{40}{2}\sin(60)=4.33\,N/mm^2$$

Member subjected to Bi-axial state of Stress

Consider a member subjected to two *tensile loads* in *x* and *y* directions resulting in tensile stresses σ_x and σ_y in *x* and *y* directions respectively as shown in figure-1.13.

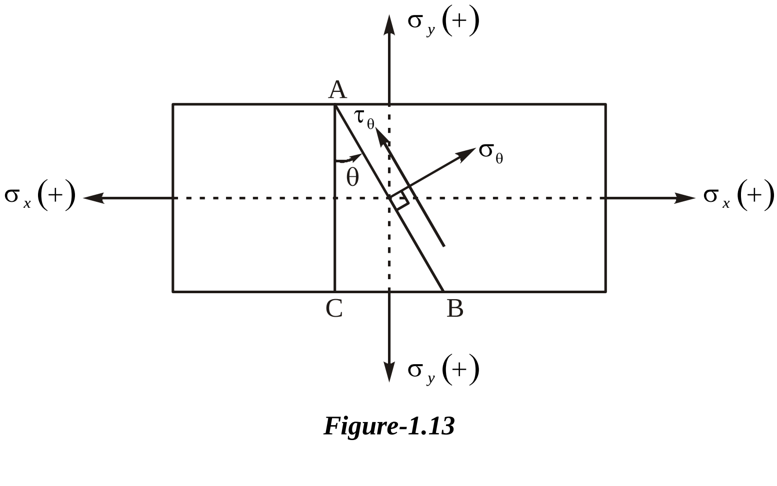

Figure-1.13

The normal stress σ_θ on a plane inclined at θ with the vertical AC is given by

Normal stress $\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$ ... E(2.26)

Shear stress $\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$ ... E(2.27)

The value of shear stress is maximum when $\theta = 45^0$

Therefore, maximum shear stress $\tau_{\max} = \left(\frac{\sigma_x - \sigma_y}{2}\right)$

Example 1.3: The principal stresses at a point in bar are $100\ N/mm^2$ (tensile) and $50\ N/mm^2$ (compressive). Compute the values of normal and tangential stresses on a plane inclined at 30^0 with the axis of major stress.

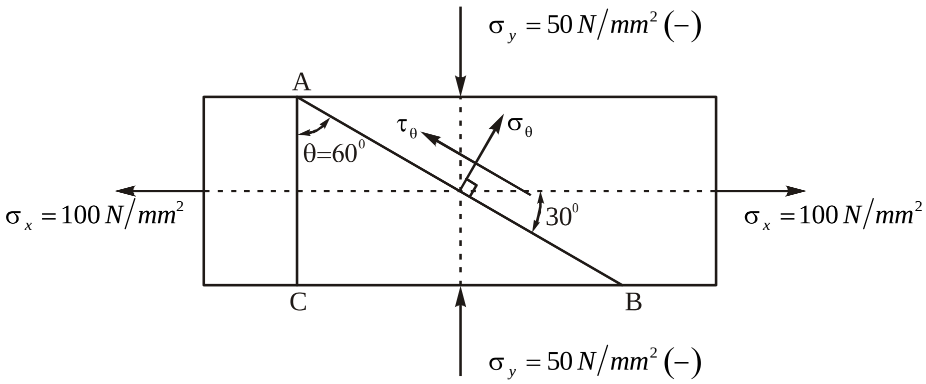

Figure-1.14

Solution:

Referring figure-1.14, $\sigma_x = 100\ N/mm^2$ (tensile), $\sigma_y = -50\ N/mm^2$ (compressive)

Therefore, Normal stress σ_θ on plane AB inclined at θ^0 with σ_x is given by

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \quad \text{... E(2.26)}$$

$$= \left(\frac{100 - 50}{2}\right) + \left(\frac{100 + 50}{2}\right)\cos 2\theta$$

Here $\left[\theta = (90 - 30) = 60^0\right]$

$$= 25 + 75\cos(120) = -12.5\, N/mm^2$$

Shear stress τ_θ on plane AB inclined at θ with AC is given by

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta \qquad \text{... E(2.27)}$$

$$= \left(\frac{100 + 50}{2}\right)\sin 2(60) = 64.95\, N/mm^2$$

Example 1.4: At a point in a stressed body, the principal stresses are σ_x = 100 N/mm^2, σ_y = 50 N/mm^2, both being positive. Determine the values of the normal and tangential stresses on a plane inclined at 45^0 with the vertical. What is the value of maximum shear stress and indicate its direction.

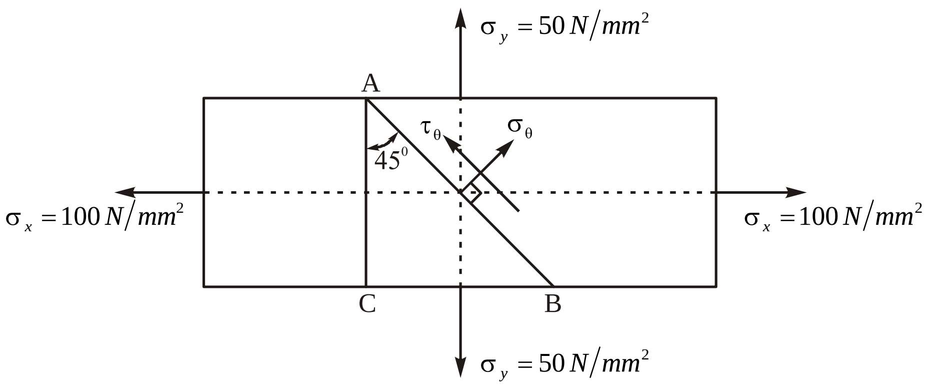

Figure-1.15

Solution:

Here, $\sigma_x = 100\, N/mm^2\,(\text{positive})$, $\sigma_y = 50\, N/mm^2\,(\text{positive})$

Normal stress σ_θ on inclined plane AB inclined at $\theta = 45^0$ with vertical AC is

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \qquad \text{... E(2.26)}$$

$$=\left(\frac{100+50}{2}\right)+\left(\frac{100-50}{2}\right)\cos(45)$$

$$=75\,N/mm^2$$

The tangential stress τ_θ on AB is

$$\tau_\theta=\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta$$

$$=\left(\frac{100-50}{2}\right)\sin 2(45)$$

$$=25\,N/mm^2$$

Maximum shear stress $\tau_{\max}$ on AB is

$$\tau_{\max}=\left(\frac{\sigma_x-\sigma_y}{2}\right) \qquad \text{... E(2.29)}$$

$$=\frac{100-50}{2}=25\,N/mm^2$$

Member subjected to Bi-axial stress accompanied with shear stress

Consider a point in a member subjected to bi-axial state of stress (σ_x,σ_y) accompanied with shear stress τ_{xy} as shown in figure-1.16.

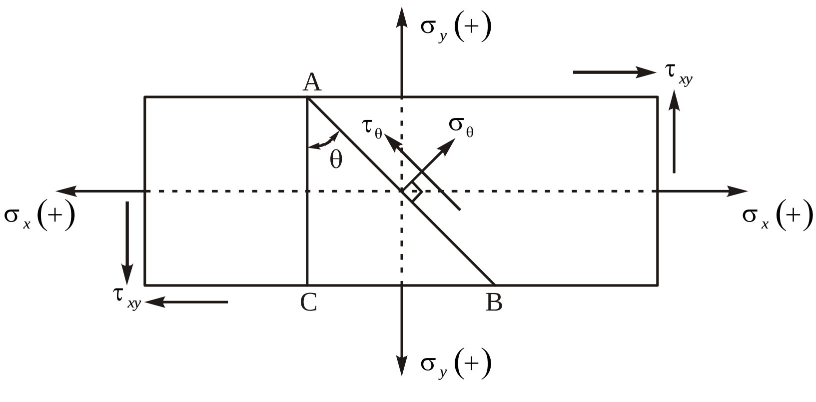

Figure-1.16

The normal stress σ_θ on a plane AB inclined at an angle θ with vertical AC is

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{... E(2.30)}$$

The shear stress τ_θ on AB is

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{... E(2.31)}$$

Example 1.5: At a point in a member, the stresses act as shown in figure-1.17. Determine the values of normal and tangential stresses on a plane inclined at 45^0 with vertical.

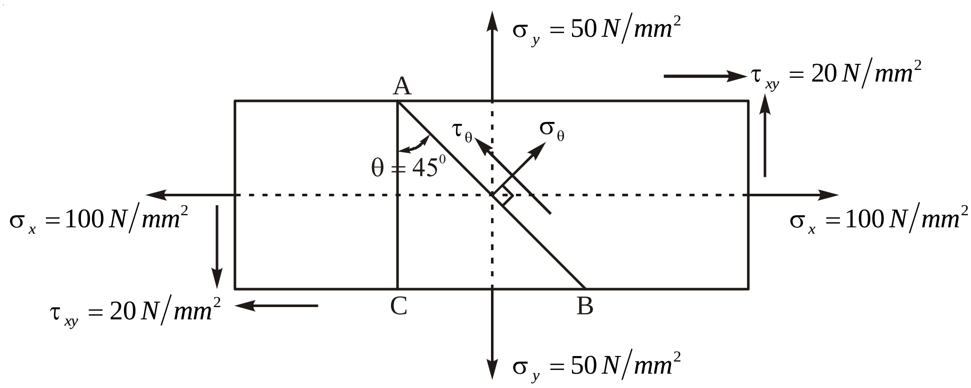

Figure-1.17

Solution:

In figure-1.17, the member is subjected to bi-axial stress state accompanied with shear.

Where, $\sigma_x = 100\, N/mm^2$ (positive)

$\sigma_y = 50\, N/mm^2$ (positive)

$\tau_{xy} = 20\, N/mm^2$ (positive)

Therefore stress σ_θ on AB

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{... E(2.30)}$$

$$\sigma_\theta = \left(\frac{100+50}{2}\right) + \left(\frac{100-50}{2}\right)\cos(2\times 45) + 20\sin(2\times 45)$$

$$= 75 + 20 = 95\,N/mm^2$$

Tangential stress τ_θ on AB is

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{... E(2.31)}$$

$$= \left(\frac{100-50}{2}\right)\sin(2\times 45) + 20\cos(2\times 45) = +25\,N/mm^2 \qquad \text{... E(2.31)}$$

Principal Stress, Principal Planes

At a point, in a stressed body there are three mutually perpendicular planes on which only normal stress exists. These three planes are called "principal planes".

The normal stress acting on the planes where shear stress is zero are called principal stresses. The principal stress occur on mutually perpendicular planes.

i) Maximum principal stress σ_1 is given by

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... E(2.32)}$$

ii) Minimum principal stress σ_2 is given by

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... E(2.33)}$$

iii) The direction of principal stress is given by

$$2\theta = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad \text{... E(2.34)}$$

iv) Maximum shear stress is given by

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Example 1.6: At point in a stressed body, the stresses act as shown in figure-1.18. Calculate the values of,

i) ***Normal and tangential stresses σ_θ and τ_θ on a plane inclined at 45^0 with the vertical.***

ii) ***The principal stresses σ_1 and σ_2.***

iii) ***The orientation of principal stresses.***

iv) ***The maximum shear stress and its direction.***

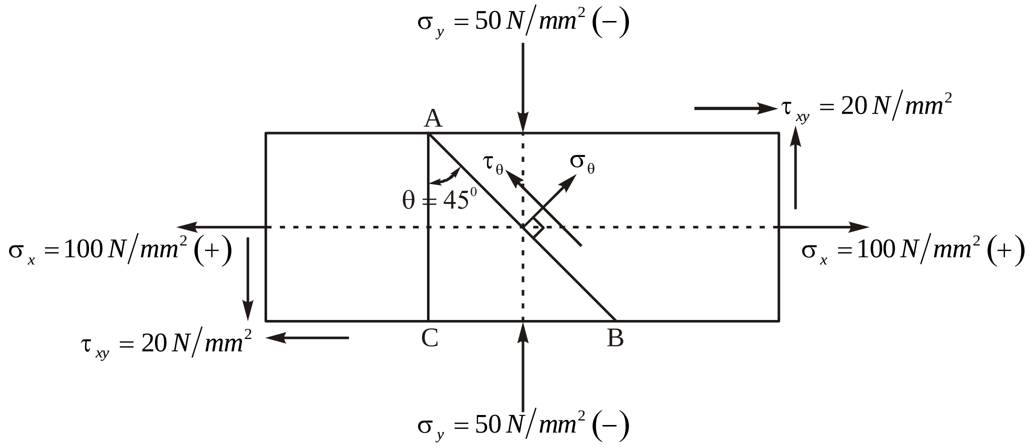

Figure-1.18

Solution:

a) ***Normal stress (σ_θ)***

Normal stress

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{... E(2.30)}$$

Here, $\sigma_x = 100\, N/mm^2$

$\sigma_y = -50\, N/mm^2$

$\theta = 45^0$

$\tau_{xy} = 20\, N/mm^2\,(\text{positive})$

Therefore, $$\sigma_\theta = \left(\frac{100-50}{2}\right) + \left(\frac{100+50}{2}\right)\cos(90) + 20\sin(90)$$

$$= 25 + 0 + 20 = 45\, N/mm^2$$

b) ***Tangential stress*** (τ_θ)

Tangential stress

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{... E(2.31)}$$

$$= \left(\frac{100+50}{2}\right)\sin(90) + 20\cos 90$$

$$\tau_\theta = 75\, N/mm^2$$

c) ***Principal stresses,*** σ_1 ***and*** σ_2

i) ***Maximum principal stress*** (σ_1)

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... E(2.32)}$$

$$= \left(\frac{100-50}{2}\right) + \sqrt{\left(\frac{100+50}{2}\right)^2 + (20)^2}$$

$$= 102.62\, N/mm^2$$

ii) ***Minimum principal stress*** (σ_2)

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{100-50}{2}\right) - \sqrt{\left(\frac{100+50}{2}\right)^2 + (20)^2}$$

$$= -52.62\, N/mm^2$$

d) ***Direction of principal stress*** (θ_1, θ_2)

$$2\theta_{1,2} = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \qquad \text{... E(2.34)}$$

$$= \tan^{-1}\left(\frac{2 \times 20}{100 - 50}\right)$$

$$= 14.93^0$$

Therefore, $\theta_1 = 7.47^0$,

$$\theta_2 = 7.47 + \frac{180}{2} = 97.47^0$$

e) ***Maximum shear stress*** $(\tau_{\max})$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... E(2.35)}$$

$$= \sqrt{\left(\frac{100 + 50}{2}\right)^2 + 20^2} = 77.62\ N/mm^2$$

f) ***Direction of shear stress*** θ_τ

$$2\theta_\tau = \tan^{-1}\left[\frac{\sigma_y - \sigma_x}{2\tau_{xy}}\right]$$

$$= \tan^{-1}\left[\frac{-50 - 100}{2 \times 20}\right] = -75.07^0$$

Therefore, $\theta_\tau = -37.53^0$

Member subjected to Tri-axial state of stress

Figure-1.19 shows a member subjected to normal stresses σ_x, σ_y and σ_z in three mutually perpendicular directions and we say that the member is in a state of Tri-axial stress.

The normal stresses in terms of normal strains in *x, y* and *z* direction are given by

$$\sigma_x = \frac{E}{(1+\gamma)(1-2\gamma)}\left[(1+\gamma)\in_x + \gamma\left(\in_y + \in_z\right)\right]$$

$$\sigma_x = \frac{E}{(1+\gamma)(1-2\gamma)}\left[(1+\gamma)\in_y +\gamma(\in_x + \in_z)\right]$$

$$\sigma_z = \frac{E}{(1+\gamma)(1-2\gamma)}\left[(1+\gamma)\in_z +\gamma(\in_x + \in_z)\right]$$

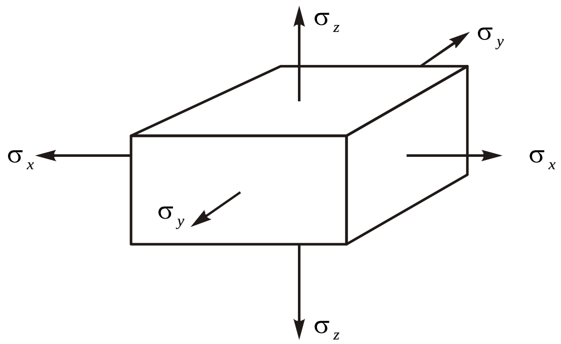

Figure-1.19

Where γ is the Poisson's ratio of the material of member and $\in_1$, $\in_2$ and $\in_3$ are the strains in *x*, *y* and *z* directions. The stress matrix is represented by the stress tensor.

$$\sigma = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{zy} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{vmatrix}$$

The maximum shear stress $\tau_{\max}$ is given by the numerical value of maximum of these values.

That is, $$\tau_{\max} = \left|\frac{\sigma_x - \sigma_y}{2}\right|$$

Or, $$\tau_{\max} = \left|\frac{\sigma_y - \sigma_z}{2}\right|$$

Or, $$\tau_{\max} = \left|\frac{\sigma_z - \sigma_x}{2}\right|$$

Definition of Stress

Stress is a concept invented by man in attempt to explain what happens within a solid body when the body is pulled, pushed, bent or twisted by external forces.

a) Axial Stress

Figure-1.20(a) shows a connecting rod of rectangular cross section subjected to a pull *F*.

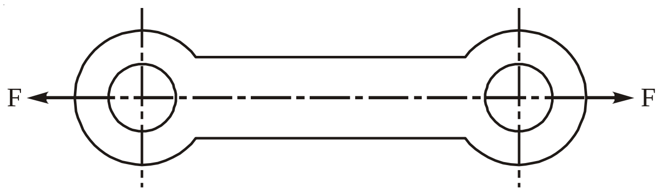

Figure-1.20(a)

Figure-1.20(b) shows the tensile force σ_{acting} on the cut portion given by $\sigma = \dfrac{F}{A}$

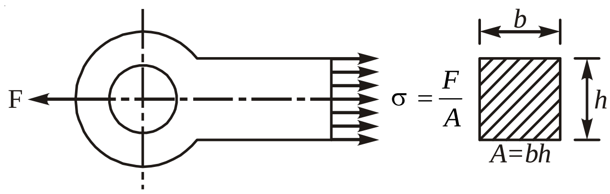

Figure-1.20(b)

In figure-1.20(c) is shown the same connecting rod subjected to push (compressive) *F*.

Figure-1.20(c)

Figure-1.20(d) shows the compressive stress σ_{acting} on the cut portion given by $\sigma = \dfrac{F}{A}$

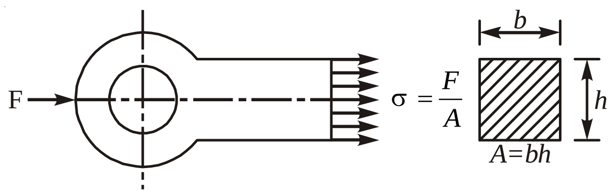

Figure-1.20(d)

In general, the formula for stress is $\sigma = \pm \frac{F}{A}$ where A is the cross sectional area of the rod.

The *positive* sign is used for *tensile stresses* and *negative sign* for *compressive stresses*. The unit of stress is N/mm^2 or MPa (Mega Pascal).

In deriving the above formula, we have assumed that the stress σ is uniform for every particle of the area on the cut portion of the rod and the stress is uniformly distributed. To have a uniform stress distribution requires that,

i) The stressed member is straight.

ii) The material is homogeneous.

iii) The line of action of force is coincident with the centroidal axis of the member.

iv) The cross section be remote from the ends of the rod and not close to any abrupt change in cross section.

The stress defined above is called *normal stress* as it acts normal to (or perpendicular to) the area of cross section.

b) Bending Stress

When a beam is bent, both shear stresses and normal stresses may occur. The shear stresses result from the shear forces. The normal stresses result from the bending moments.

In developing beam theory, we assume that the material is Isotropic and homogeneous. This means that the material must have the same properties in all directions. Cast iron and concrete are examples of materials that are neither Isotropic nor homogeneous. They are not homogeneous because they contain inclusions have different physical characteristics than the matrix. They are not isotropic because the modulus of elasticity is different in tension and in compression.

We also require that the beam to be straight. Another requirement is that the material must obey Hooke's law. i.e., the material of the beam must be elastic, have a straight stress-strain line, and the beam not to be bent so much so as to result in a stress greater than the elastic limit.

The analysis of beams in this book is restricted to beam having constant cross section throughout the length and at the same time have an axis of symmetry in the plane of bending.

I, H and O section are some of the sections that satisfy the symmetry requirement. Finally, the plane cross sections in a beam under pure bending remain plane during bending.

The equation connecting bending moment M_b, moment of Inertial '*I*' and bending stress σ_b is given by

$$\frac{M_b}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

Where, $M_b =$ Bending moment; N-mm

$\sigma_b =$ Bending stress; N/mm^2

$I =$ Moment of Inertia; mm^4

$E =$ Modulus of Elasticity; N/mm^2

$R =$ Radius of curvature; mm

$y =$ Distance of neutral axis from the extreme fibre; mm

$$\sigma_b = \frac{M_b y}{I}$$

$$y = c = \frac{Depth\,of\ beam}{2}$$

Therefore, $$\sigma_b = \frac{M_b}{\left(\frac{I}{c}\right)}$$

The quantity $\frac{I}{c}$ is called section modulus Z_b

Therefore, $$\sigma_b = \frac{M_b}{Z_b}$$

Note: For circular cross section,

$$Z_b = \frac{I}{c} = \frac{\left(\frac{\pi d^4}{6^4}\right)}{\left(\frac{d}{2}\right)} = \frac{\pi d^3}{32} \qquad \text{... T(2.6)}$$

And for hollow section,

$$Z_b = \frac{I}{c} = \frac{\pi\left(d_0^4 - d_i^4\right)}{32 d_0} \qquad \text{... T(2.6)}$$

c) *Torsional Stress*

A moment that cause twisting of a bar or a rod is called torque. The external twisting moment M_t or external torque may be designated by drawing arrows on the surface of the bar to indicate the direction only. The analysis in this book is restricted to torsion of circular rods (solid and hollow type).

The equation of torque M_t relating angular deflection θ, polar moment of inertia J is given by

$$\frac{M_t}{J} = \frac{G\theta}{l} = \frac{\tau}{r}$$

Where, $M_t =$ Twisting moment of torque; N-mm

$J =$ Polar moment of Inertia; mm^4

$J = \frac{\pi d^4}{32}, mm^4$ for circular cross section

$= \frac{\pi\left(D^4 - d^4\right)}{32}$ for hollow shaft having outside diameter D and inside diameter d

$\theta =$ Total angle of twist of beam in radians

$l =$ Length of beam; mm

$G =$ Modulus of rigidiry; N/mm^2

$\tau =$ Shear stress; N/mm^2

$r =$ Radius of shaft; mm

From the above equation,

$$\frac{M_t}{J} = \frac{\tau}{r}$$

i.e.,

$$\tau = \frac{M_t r}{J} = \frac{M_t}{\left(\frac{J}{r}\right)}$$

Where, $\frac{J}{r} =$ Section modulus in torsion $= \frac{\pi d^3}{16}$ ... T(2.1)

$$= \frac{J}{c}$$

Substituting $r = \frac{d}{2}$, $J = \frac{\pi d^4}{32}$, we get

$$\tau = \frac{M_t\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16M_t}{\pi d^3}$$

In computing shear stress τ in rotating shafts, it is necessary to obtain torque M_t from the consideration of *N*, the power transmitted in kilowatts and the shaft speed *n* given by

$$M_t = \frac{9550N}{n}\text{ N-m}$$

$$= \frac{9550N}{n} \times 10^3 \text{ N-mm}$$

Where, *N* = Power transmitted in kilowatts.
n = Speed of shaft in rpm.

Static Loading:

Static loading may be defined as load which is applied to a machine part very gradually increasing from zero to its maximum value. The application of load is so gradual that the acceleration occurring can be neglected.

After the load reaches its final value, it does not change in magnitude, direction or point of application with time.

Factor of Safety

In designing any machine component it is necessary to ensure that the stress induced in the component is less that or equal to the design stress or allowable stress.

The present definition of factor of safety is the amount by which the design stress is kept below the limit stress. Limit stress is the maximum stress to which a machine component can be subjected without being fractured.

For ductile materials subjected to static loading, the maximum stress is the yeild stress and hence

$$Factor\ of\ safety = \frac{Yield\ stress}{Design\ stress} = \frac{\sigma_y}{\sigma_d}$$

For Brittle materials, the maximum stress is the ultimate stress of the material and hence

$$Factor\ of\ safety = \frac{Endurance\,limit}{Design\,stress} = \frac{\sigma_{-1}}{\sigma_d}$$

Values of Factor of Safety

Factor of safety = 1.25 to 1.5 for exceptionally reliable materials.
= 1.5 to 2 for well known materials.
= 2 to 2.5 for average materials.
= 2.5 to 3 for less tried materials.
= 3 to 4 for untried materials.

Theories of Failure

In designing any machine part, it is necessary to ensure that the stresses induced in the member do not exceed the strength of material. In case of ductile materials, yield strength is considered whereas in case of brittle materials. Ultimate strength is considered as the criterion of failure. Ductile materials are equally strong in tension and compression where as brittle materials are stronger in compression than in tension. Steel is an example for ductile material and cast iron for brittle material. Various theories have been developed to understand and predict the failure criteria. Amongst these the following two theories are very important.

a) Distortion Energy Theory (Von-mises Henky Theory)

This theory states that failure occurs when the strain energy of distortion per unit value at any point becomes equal to the strain energy of distortion per unit volume in a standard tension test specimen, when yielding starts.

This theory is best suited for ductile materials and gives accurate results.

In this tension test,

$$\sigma_1 = \sigma_x = \sigma_{yt}; \qquad \sigma_2 = \sigma_3 = 0$$

For a tri-axial stress state, we have

$$\left(\frac{\sigma_{yt}}{Factor\ of\ safety}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1$$

For a two dimensional case, $\sigma_3 = 0$

Therefore, $$\left(\frac{\sigma_{yt}}{Factor\ of\ safety}\right)^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$= \sigma_x^2 + \sigma_y^2 + 3\tau_{xy}^2$$

b) *Maximum Normal Stress Theory (or Rankine Theory)*

According to this theory, yielding at a point occurs only when the maximum principal stress on any plane passing through that point reaches a value equal to the elastic limit as computed in simple tension test.

i.e., $\sigma_{\max} = \sigma_y$

If the largest positive principal stress is designated as $\sigma_{\max}$ (tensile) and larger negative principal stress is take as $\sigma_{\max}$ (compressive) then

$$\sigma_{\max} \le Allowable\ stress$$

$$\sigma_{\max}(tensile) \le \frac{\sigma_{yt}}{Factor\ of\ safety}$$

$$\sigma_{\max}(compressive) \le \frac{\sigma_{yc}}{Factor\ of\ safety}$$

Note: When the material has no distinct yield point, ultimate stress in tension and compression may be used in place of yield point stress.

Therefore,

$$\sigma_{\max}(tensile) \le \frac{\sigma_{ut}}{Factor\ of\ safety}$$

$$\sigma_{\max}(compressive) \le \frac{\sigma_{uc}}{Factor\ of\ safety}$$

Failure of Brittle Materials

i) Failure of brittle materials takes place by fracture.

ii) Brittle materials do not have a distinct yield point and so the ultimate strength is used as the basis for determining the allowable or design stress.

iii) Separate design equations should be used in tension and compression since brittle materials have a considerable higher value of ultimate compressive strength than the ultimate tensile strength.

iv) Maximum principal stress theory is used for the materials.

Note:

a) If both principal stresses are of same sign, the effect of smaller stress is neglected and hence

$$\sigma_1 \le \frac{\sigma_{ut}}{Factor\ of\ safety}(\ for\ tension)$$

$$\sigma_1 \le \frac{\sigma_{uc}}{Factor\ of\ safety}(\ for\ compression)$$

b) ***If both principal stresses are of opposite sign, then the maximum principal stress theory does not give conservative results. In such case "Coulomb-Mohr Theory" given by***

$$\frac{\sigma_1}{\sigma_{ut}} + \frac{\sigma_2}{\sigma_{ut}} = \frac{1}{Factor\ of\ safety}$$ is used.

Failure of Ductile Materials

i) Ductile materials fail by yielding.

ii) The working or allowable stress is based on yield point stress.

iii) Maximum shear stress theory is used in design as it gives conservative results.

In general, machine elements depending on the application are subjected to the following types of loads.

a) Tensile Load

Consider a rectangular plate of width *b* and thickness *h* subjected to tensile load *F*. The tensile stress due to load *F* at cross section A-A is given by

$$\sigma_1 = \frac{F}{Area\ of\ cross\ section} = \frac{F}{bh}$$

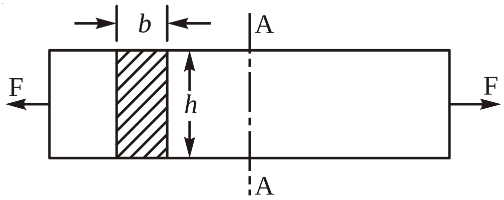

Figure-1.20(e)

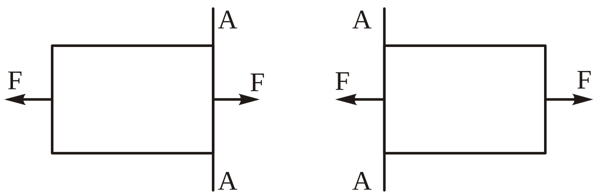

Figure-1.20(f)

Figure-1.20: Rectangular Plate in Tension

Figure-1.21 shows a stepped shaft with smallest diameter '*d*' subjected to tensile load *F*. The least diameter *d* is subjected to maximum tensile stress given by

$$\sigma_t = \frac{F}{Area\ of\ cross\ section} = \frac{F}{\frac{\pi d^2}{r}} = \frac{4F}{\pi d^2}$$

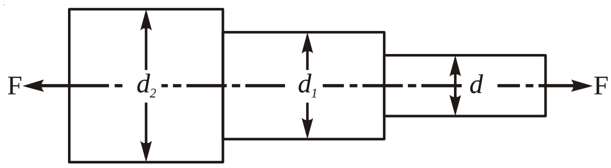

Figure-1.21

Figure-1.22 shows an eye bolt provided on the body of electric motors for the purpose of lifting.

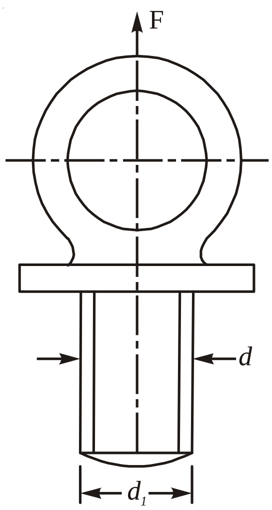

Figure-1.22

The threads in the bolt portion having core diameter d_1 are subjected to tensile load and hence tensile stress in the screwed rod is given by

$$\sigma_t = \frac{F}{A_c}$$

Where, $A_c = \text{core area} = \frac{\pi d_1^2}{4}$

$$\sigma_t = \frac{F}{\frac{\pi d_1^2}{4}} = \frac{4F}{\pi d_1^2}$$

Knowing tensile stress σ_t, and load *F*, the core diameter d_1 can be calculated.

b) *Compressive Load*

In figure-1.23 is shown a steel tube of outside diameter *D* and inside diameter *d* subjected to an axial compressive load *F*.

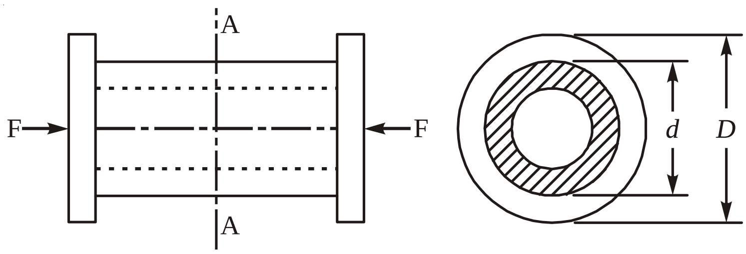

Figure-1.23

The compressive stress σ_c at cross section A-A due to load *F* is given by

$$\sigma_c = \frac{F}{Area\ of\ cross\ section} = \frac{F}{\frac{\pi}{4}\left(D^2 - d^2\right)} = \frac{4F}{\pi\left(D^2 - d^2\right)}$$

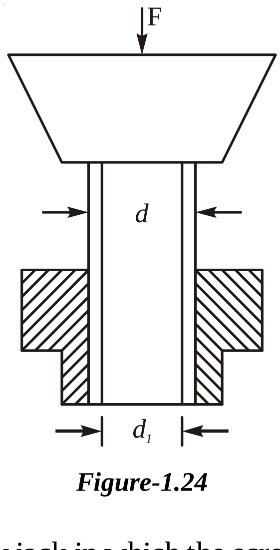

Figure-1.24

Figure-1.24 shows a screw jack in which the screwed rod having basic diameter (or nominal diameter) d and core diameter d_1 subjected to an axial compressive load F (i.e., load being raised)

The compressive stress at the core area due to F is given by

$$\sigma_c = \frac{F}{A_c} = \frac{4F}{\frac{\pi d_1^2}{4}} = \frac{4F}{\pi d_1^2}$$

Where, A_c is the core area $= \frac{\pi d_1^2}{4}$

c) *Shear Load*

i) *Direct shear*

In direct shear loading, the load acts parallel to the area under shear.

Figure-1.25 shows a rivet of diameter d in a riveted lap joint failing by single shear due to load F.

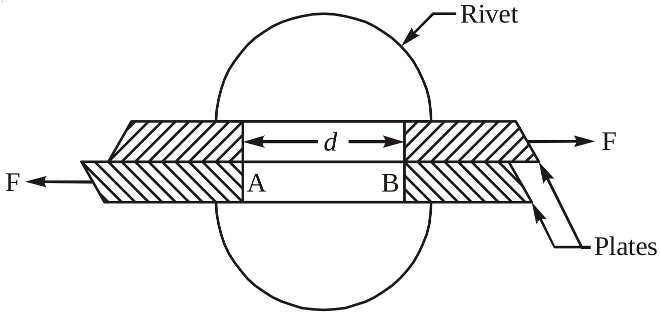

Figure-1.25(a)

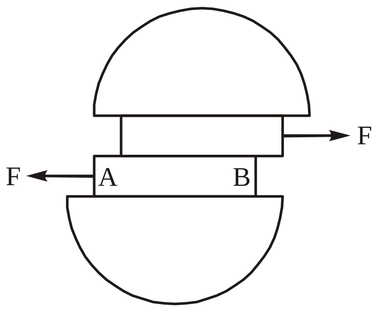

Figure-1.25(b)

Figure-1.25: Rivet under Single Shear

Therefore, Direct shear stress $\tau = \frac{F}{Area\ of\ cross\ section} = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$

Figure-1.26 shows a rivet of diameter *'d'* failing by double shear due to load F. The shear stress due to double shear is

$$\tau = \frac{F}{Area\ of\ cross\ section} = \frac{F}{2\left[\frac{\pi d^2}{4}\right]} = \frac{2F}{\pi d^2}$$

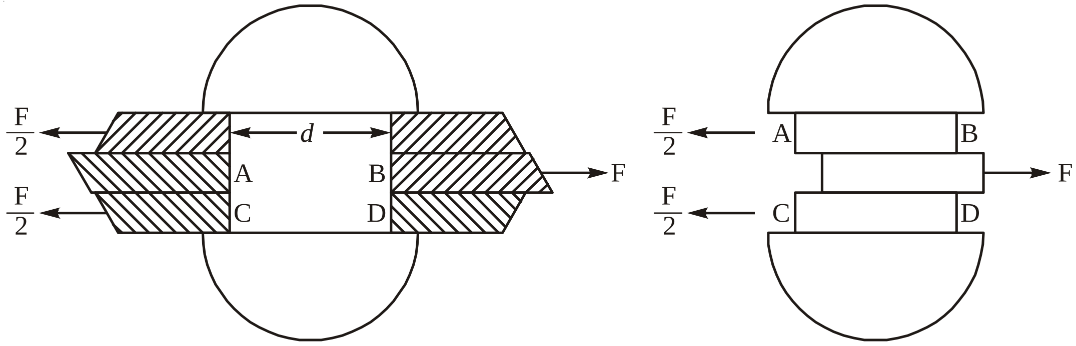

Figure-1.26: Rivet under Double Shear

ii) Torsional Shear Load

When a machine element is subjected to twisting moment or torque M_t, torsional shear stress is induced on the surface of member and is given by

$$\tau = \frac{M_t}{Z_t}$$

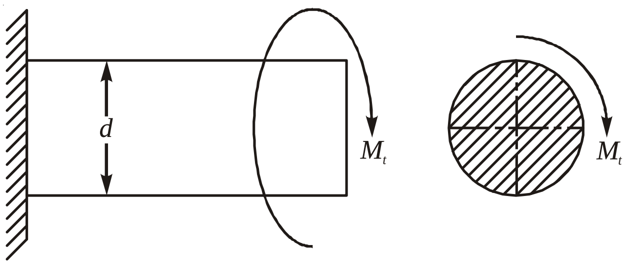

Figure-1.27

Where, $\tau = Torsional\ shear\ stress$

$$Z_t = Section\ modulus\ in\ torsion = \frac{J}{c}$$

$$Z_t = Section\ modulus = \frac{\pi d^3}{16} \qquad \text{... T(2.2)}$$

$d = Shaft\ diameter$

Therefore, $$\tau = \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16 M_t}{\pi d^3}$$

Figure-1.28 shows a hallow shaft having inside diameter *d* and outside diameter *D* subjected to torque M_t.

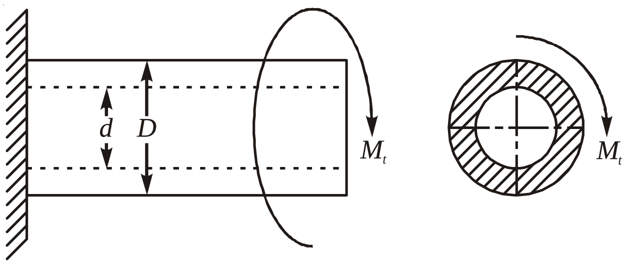

Figure-1.28

The torsional shear stress induced on the outer surface of the shaft is given by

$$\tau = \frac{M_t}{Z_t} = \frac{M_t}{\frac{\pi\left(D^4 - d^4\right)}{16D}} = \frac{16M_tD}{\pi\left(D^4 - d^4\right)}$$

d) ***Bending Loads***

A shaft of diameter *d* subjected to bending moment M_b is shown in figure-1.29.

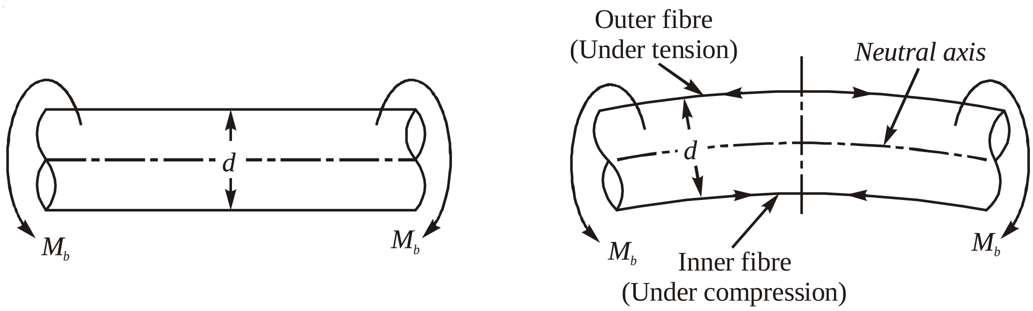

Figure-1.29

Due to M_b, bending stress (tensile) is induced in the outer fibre and bending stress (compressive) is induced in the inner fibre given by

$$\sigma_b = \frac{M_b}{Z_b} \qquad \text{... T(2.7)}$$

Where, $Z_b = \frac{I}{c} = \textit{section modulus} = \frac{\pi d^3}{32}$

Therefore, Bending stress $\sigma_b = \dfrac{M_b}{\dfrac{\pi d^3}{32}} = \dfrac{32M_b}{\pi d^3}$ (Tensile at the outer fibre and compressive at the inner fibre)

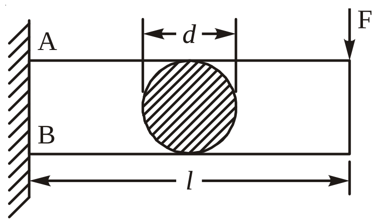

Figure-1.30

Figure-1.30 shows a cantilever beam of length '*l*' subjected to a point load *F* at its end. Due to *F*, the bending stress (Tensile at A and compressive at B) induced in the beam is given by

$$\sigma_b = \frac{M_b}{Z_b} \qquad \text{... T(2.7)}$$

Where, $M_b = Fl$ and Section modulus $Z_b = \dfrac{I}{c}$

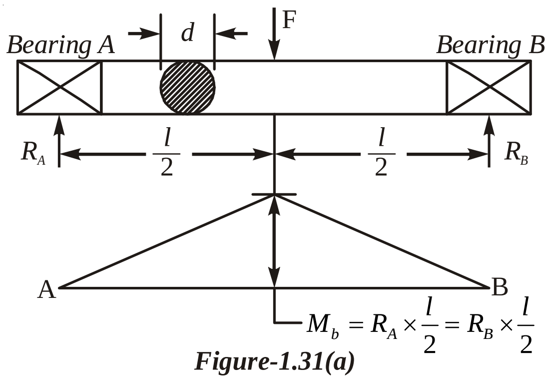

Figure-1.31(a)

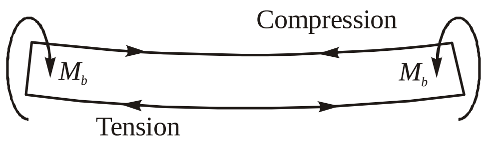

Figure-1.31(b)

Figure-1.31(a) shows a beam of length '*l*' simply supported between A and B subjected to point load *F*.

Due to *F,* tensile bending stress is induced in the lower fibre and compressive bending stress in the upper fibre as shown in figure-1.31(b).

The magnitude of bending stress is given by

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = R_A \times \frac{l}{2}$

Because of symmetry $R_A = R_B = \frac{F}{2}$

$$= \frac{Fl}{4}$$

and $$Z_b = Section\ modulus = \frac{I}{c} = \frac{\pi d^3}{31}$$

Therefore, $$\sigma_b = \frac{\frac{Fl}{4}}{\frac{\pi d^3}{32}} = \frac{Fl}{4} \times \frac{32}{\pi d^3} = \frac{8Fl}{\pi d^3}$$

Bearing Load

Figure-1.32 shows a journal bearing of diameter *'d'* and length *'l'* supporting a load *F*.

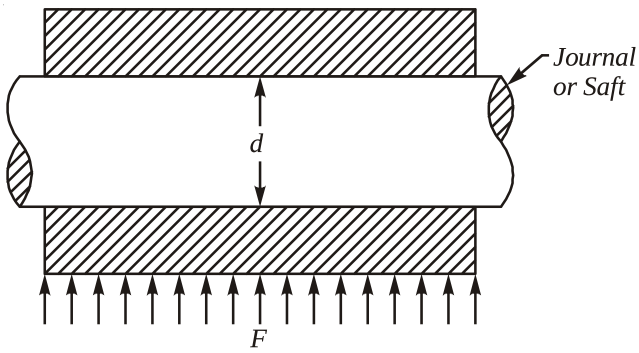

Figure-1.32

The bearing pressure P_b induced in the bearing due to *F* based on projected area of bearing is given by

Therefore, Bearing pressure $P_b = \frac{Bearing\ load}{Projected\ are\ of\ bearing} = \frac{F}{ld}$

Example 1.7: A link in figure-1.33 is required to transmit a tensile load of 60kN. Determine the stress induced in the link at sections 1-1 and 2-2.

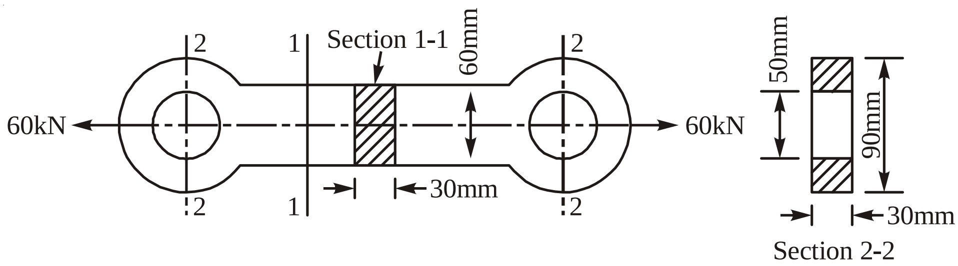

Figure-1.33

Solution:

Due to load, tensile stress is induced at sections 1-1 and 2-2

At section 1-1,

$$\text{Tensile stress,}\quad \sigma_{t(1-1)} = \frac{Load}{Area\ of\ cross\ section\ 1\text{-}1} = \frac{60kn}{60\times 30}$$

$$= \frac{60\times 10^3}{60\times 30} = 33.33\, N/mm^2$$

At section 2-2,

$$\text{Tensile stress,}\quad \sigma_{t(2-2)} = \frac{Load}{Area\ of\ cross\ section\ 2\text{-}2}$$

$$= \frac{60\times 10^3}{\left[(90\times 30)-(50\times 30)\right]} = 50\, N/mm^2$$

Example 1.8: A link shown in figure-1.34 by solid line is required to transmit a pull of 100kN. Determine the dimensions b and h taking $b = 3h$, permissible stress equal to 80MPa. If the original link is replaced by a link of small thickness h shown by dotted lines, determine the depth B.

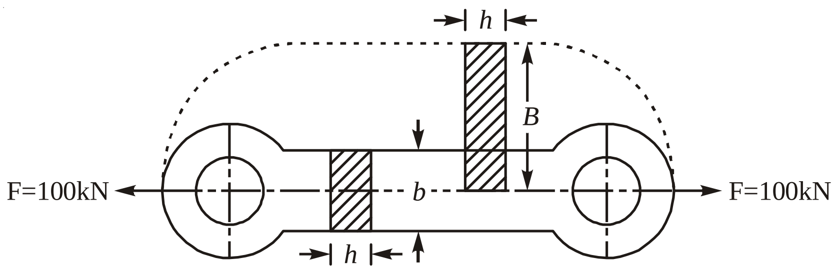

Figure-1.34

Solution:

The link is subjected to tensile stress due to F. For the original link having width h and depth b,

$$\sigma_{tensile} = \frac{F}{Area\ of\ cross\ section} = \frac{F}{hb} = \frac{F}{h \times 3h}$$

$$= \frac{100 \times 10^3}{3h^2} = \frac{33.33 \times 10^3}{h^2}$$

But, $\sigma_{tensile} = 80MPa$

Therefore, $80 = \dfrac{33.33 \times 10^3}{h^2}$

i.e., $h = 20.41mm$ and hence $B = 3h = 3 \times 20.41 = 61.23mm$

When the cross section is changed to h and B, the link is subjected to both tensile stress and bending stress (due to eccentricity).

Tensile stress $\sigma_t = \dfrac{F}{Area\ of\ cross\ section} = \dfrac{100 \times 10^3}{hB}$

$$= \frac{10^5}{20.41 \times B} = \frac{4899.6}{B} N/mm^2$$

Bending stress due to eccentricity in loading is

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = F\left(\dfrac{B}{2}\right) = \dfrac{100 \times 10^3 \times B}{2} = 50 \times 10^3 B$ N-mm

$$Z_b\left(Section\ modulus\right) = \frac{I}{c}$$

Where, $c = \dfrac{B}{2}$

$$Z_b = \frac{\left(\frac{hB^3}{12}\right)}{\frac{B}{2}} = \frac{hB^2}{6} = 20.41\left(\frac{B^2}{6}\right) = 3.4B^2$$

i.e., $\sigma_b = \dfrac{M_b}{Z_b}$ becomes

$$\sigma_b = \frac{50\times10^3 B}{3.4B^2} = \frac{14705.9}{B}$$

Now, $\sigma = \sigma_t + \sigma_b$

$$80 = \frac{4899.6}{B} + \frac{14705.9}{B}$$

Therefore, $B = 245.06mm$

Therefore, the depth B of the replaced link = 245.06mm.

Example 1.9: A knuckle joint is used to connect shafts of diameter D to transmit a tensile load of 5kn. Determine the diameter of the shaft and the pin required using permissible tensile stress in the shaft as 60MPa and the shear stress in pin as 40MPa.

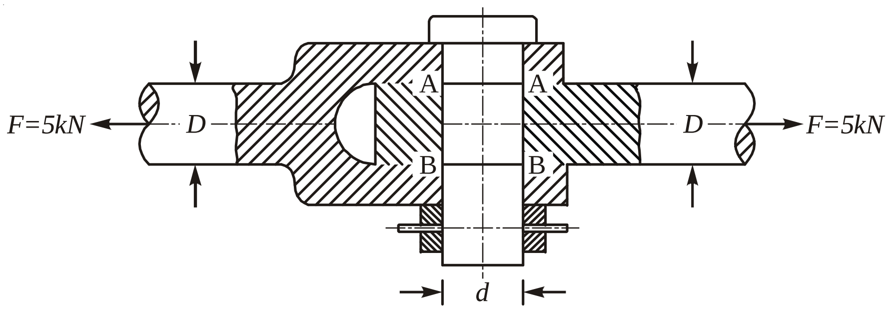

Figure-1.35

Solution:

The rod is subjected to tensile load F.

Therefore, Tensile stress in the rod due to $F = \sigma_t = \dfrac{F}{A}$

Where, $F = 5kN = 5\times10^3 N$

$$A = \frac{\pi D^2}{4}$$

From, $\sigma_t = \dfrac{F}{A}$,

i.e., $$60 = \frac{5 \times 10^3}{\frac{\pi D^2}{4}}$$

Therefore, $D = 10.3mm.$

Diameter of the rod $D = 10.3mm.$

The pin having diameter *'d'* is subjected to double shear at sections A-A and B-B and hence the shear stress τ in the pin is

$$\tau = \frac{F}{2[Area\, of\ cross\ section\, of\ pin]}$$

$$= \frac{F}{2\left(\frac{\pi}{4}d^2\right)} = \frac{2F}{\pi d^2}$$

Substituting the values,

$$40 = \frac{2 \times 5 \times 10^3}{\pi d^2}$$

d = Diameter of pin = 8.92mm.

Example 1.10: A steel bracket of rectangular section is loaded as shown in the figure-1.36. Determine the thickness h at section A-A by limiting the tensile stress to $80\, N/mm^2$.

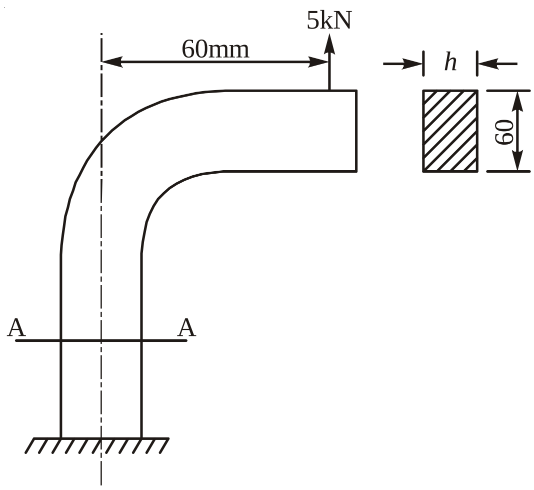

Figure-1.36

Solution:

Load on the cross section, $F = 5kN = 5\times10^3 N$

Therefore, Tensile stress $= \dfrac{F}{A} = \dfrac{5\times10^3}{60h} = \dfrac{83.33}{h}$

Bending stress due to F is given by,

$$\sigma_b = \frac{M_b}{Z_b}$$

i.e., $$\sigma_b = \frac{M_b c}{I}$$

Because, $$Z_b = \frac{I}{c}$$

Where, $$M_b = F\times 60 = 5\times10^3\times 60 \text{ N-mm}$$

$$= 300\times10^3 \text{ N-mm}$$

$$c = \frac{Depth}{2} = \frac{60}{2} = 30mm$$

$$I = \frac{hb^3}{12} = \frac{h}{12}(60)^3 = 18000h$$

Therefore, $$\sigma_b = \frac{300\times10^3\times30}{18000h} = \frac{500}{h}$$

Therefore, Total tensile stress at section

$$A - A = \sigma_t + \sigma_b = \frac{83.33}{n} + \frac{500}{h} = 80$$

Therefore, $h = thickness\ of\ bracket = 7.354mm.$

Example 1.11: A steel bracket shown in figure-1.37 is subjected to a load of 5kN. Determine the maximum stress induced in the member at section A-A taking diameter as 30mm.

Solution:

Tensile stress due to 5kN is

$$\sigma_t = \frac{Load}{Area\ of\ cross\ section}$$

$$= \frac{5kN}{\frac{\pi d^2}{4}} = \frac{5\times10^3}{\frac{\pi}{4}(30)^2} = 7.07\,N/mm^2$$

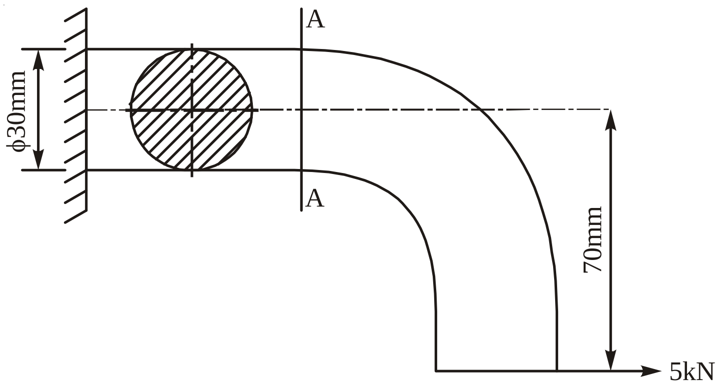

Figure-1.37

Bending stress due to load 5kN is

$$\sigma_b = \frac{M_b}{Z_b} \text{ and } Z_b = \frac{I}{c}$$

Therefore, $\sigma_b = \frac{M_b c}{I}$

Where, $M_b = \text{Bending moment}$

$$= 5\times10^3 \times 70 = 350\times10^3 \text{ N-mm}$$

$$c = \frac{Depth}{2} = \frac{d}{2} = \frac{30}{2} = 15mm$$

$$I = \frac{\pi d^4}{64} = \frac{\pi}{64}(30)^4 = 39760.78\,mm^4$$

Therefore, $\sigma_b = \frac{350\times10^3\times15}{39760.78} = 132.4\,N/mm^2$

i.e., Maximum stress induced

$$= \sigma_t + \sigma_b$$

$$= 7.07 + 132.04 = 139.11\,N/mm^2$$

Example 1.12: Determine the value of b in the cross section of a machine frame loaded as shown in figure-1.38, limiting the maximum normal stress induced in the cross section to 120MPa. The loading is through a forked bar whose fork is connected to the frame by a pin under double shear. Determine the diameter of the pin limiting the shear stress induced in it to 80MPa.

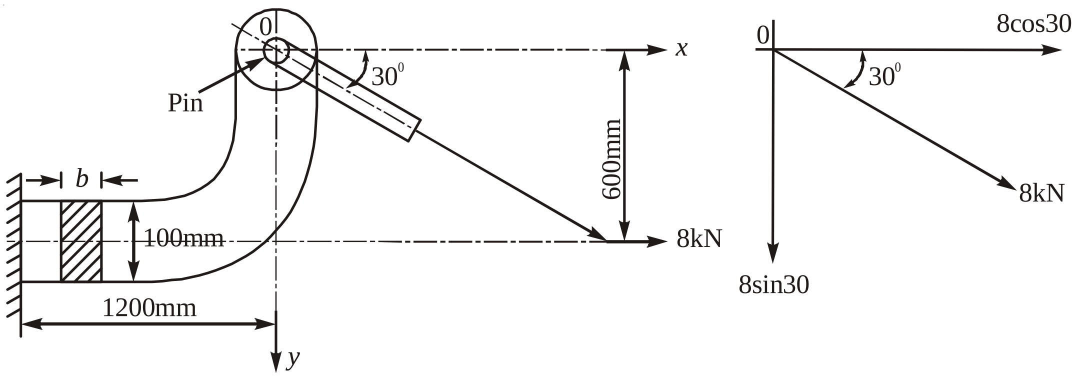

Figure-1.38

Solution:

Resolve inclined load 8kN along the horizontal and vertical directions. The horizontal component of load 8kN

$$8kN = F_h = \left(8\times10^3\right)\cos 30$$

$$= 6928.2N$$

And the vertical component of load 8kN

$$8kN = \left(8\times10^3\right)\sin 30$$

$$= 4000N$$

The horizontal component induces tensile stress at the upper surface and compressive stress at the bottom surface of frame.

The bending moment due to horizontal load F_h

$$= M_{bH} = F_h \times 600$$

$$= 6928.2\times 600 \text{ N-mm}$$

$$= 4156920 \text{ N-mm}$$

σ_{bH}, the bending stress due to M_{bH} is

$$\sigma_{bH} = \frac{M_{bH}}{Z_b}$$

Where, $Z_b = \frac{I}{c}$

$$= \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6} = \frac{b(100)^2}{6}; \quad h = 100mm$$

Therefore, $\sigma_{bH} = \frac{4156920}{\frac{b(100)^2}{6}} = \frac{2494.152}{b} N/mm^2$

σ_d the direct stress due to vertical load $= \frac{Vertical\ load}{Area\ of\ cross\ section}$

$$\sigma_d = \frac{4000}{b \times 100} = \frac{40}{b} N/mm^2$$

Bending moment due to vertical load F_v

$$M_{bv} = F_v \times 1200$$
$$= 4000 \times 1200$$
$$= 48 \times 10^5 \text{ N-mm}$$

Therefore, Bending stress due to M_{bv} is given by

$$\sigma_{bv} = \frac{M_{bv}}{Z_b}$$

Where, $Z_b = \frac{100(b)^2}{6} = 16.67b^2$

$$\sigma_{bv} = \frac{48 \times 10^5}{16.67b^2} = \frac{287942.41}{b^2}$$

Therefore, total tensile stress at the upper surface of bracket is

$$= \sigma_{bH} + \sigma_d + \sigma_{bv} = 120$$

$$\frac{2494.152}{b} + \frac{40}{b} + \frac{287942.41}{b^2} = 120$$

$$\frac{2534.152}{b}+\frac{28942.41}{b^2}=120$$

$$2534.152b+287942.4=120b^2$$

i.e., $$21.12b+2400=b^2$$

$$b^2-21.12b-2400=0$$

$$b=\frac{21.12\pm\sqrt{(21.12)^2+4\times2400}}{2}$$

$$=60.68mm$$

Therefore, width of frame = 60.68mm.

To find the pin diameter (d)

The pin is under double shear and hence the shear stress τ induced in pin is given by

$$\tau=\frac{Load}{Area\ under\ shear}=\frac{Load}{2\left(\frac{\pi d^2}{4}\right)}$$

$$\tau=\frac{8\times10^3}{2\left(\frac{\pi d^2}{4}\right)}=80$$

Therefore diameter of pin $d=7.97mm$; Use $d=8mm.$

Example 1.13: A bracket shown in figure-1.39 is subjected to a load F. Determine the load F by limiting the maximum stress in the bracket to 80MPa.

Solution:

The load F can be resolved into vertical and horizontal components.

The horizontal component of $F=F_h=F\sin30$

The vertical component of $F=F_v=F\cos30$

The bending moment M_{bH} due to F_h is

$$M_{bH}=F\sin30\times50=25F \text{ N-mm}$$

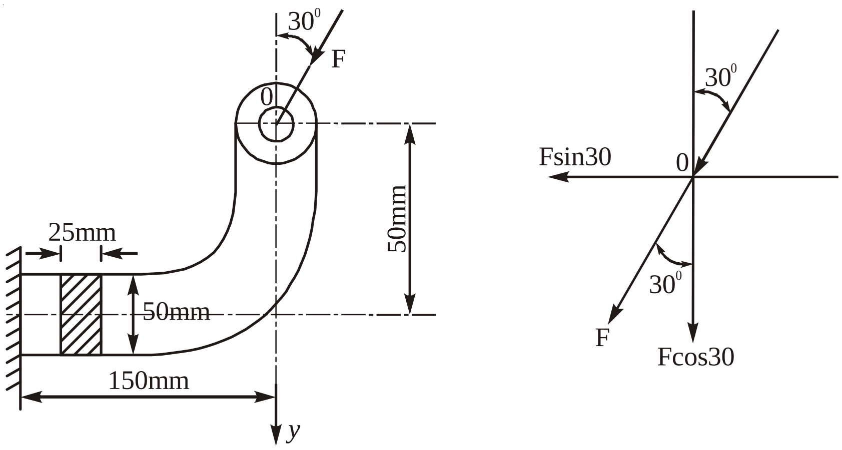

Figure-1.39

Section modulus, $Z_b = \frac{I}{c} = \frac{bh^2}{6}$

$$= \frac{25 \times 50^2}{6} = 10416.67 mm^2$$

Bending stress σ_{bH} due to M_{bH} is

$$\frac{M_{bH}}{Z_b} = \frac{25F}{10416.67} = 2.4 \times 10^3 F$$

Note: This σ_b causes tensile stress in the bottom fibre and compressive stress in the upper fibre.

The bending moment due to vertical component F_v

$$M_{bv} = F_v = F\cos 30 \times 150$$

$$= 150F\cos 30$$

Bending stress due to M_{bv} is given by

$$\sigma_{bv} = \frac{M_{bv}}{Z_b}$$

$$= \frac{150 \times F\cos 30}{10416.67} = 0.125F$$

Note: This vertical component causes tensile stress at the upper fibre and compressive stress at the inner fibre.

The direct stress due to vertical load

$$\sigma_d = \frac{F\cos 30}{Area\ of\ cross\ section}$$

$$= \frac{F\cos 30}{50\times 25} = 9.93\times 10^{-4}F$$

Therefore, the total stress in the upper fibre is

$$-\sigma_{bH} + \sigma_d + \sigma_{bv} = 80$$

$$-2.4\times 10^{-3}F + (6.93\times 5)10^{-4}F + 0.125F = 80$$

Therefore F, the load the bracket can withstand is = 648.9N.

Example 1.14: A circular rod of diameter 50mm shown in figure-1.40 is subjected to a point load of 3kN at its end and torque 300N-m. Determine the stresses induced in the rod.

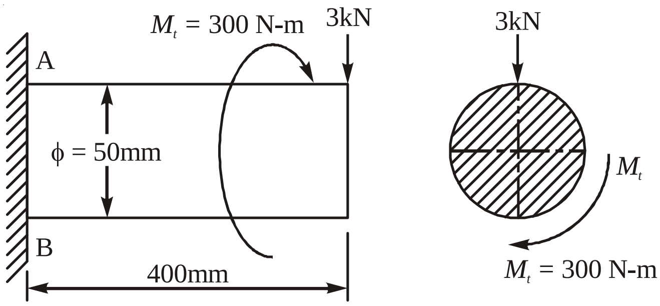

Figure-1.40

Solution:

The vertical load 3kN causes bending and hence induces bending stress in the rod.

The torque M_t causes torsional shear stress.

Now bending stress due to 3kN is given by

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = 3kN\times 400$

$$= 12\times 10^5 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

Where, $I = \frac{\pi d^4}{64}$ and $c = \frac{d}{2}$

Therefore, $Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$

$$= \frac{\pi}{32}(50)^3 = 12271.85 mm^3$$

Therefore, $\sigma_b = \frac{12 \times 10^5}{12271.85} = 97.78\, N/mm^2$

$$\sigma_b \; at \; A = +97.78\, N/mm^2$$

$$\sigma_b \; at \; B = -97.78\, N/mm^2$$

Torsional shear stress due to torque M_t is

$$\tau = \frac{M_t}{Z}$$

Where, $M_t = 300$ N-m $= 300 \times 10^3$ N-mm

$$Z_t = \frac{J}{c} = \frac{\pi d^3}{16}$$

$$= \frac{\pi}{16}(50)^3 = 24543.7\, mm^3$$

$$\tau = \frac{300 \times 10^3}{24543.7} = 12.22\, N/mm^2$$

Note: The magnitude of torsional shear stress τ is same at points A and B and is equal to $12.22\, N/mm^2$.

To find maximum shear stress

Maximum shear stress $\tau_{\max}$ is given by

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}; \qquad \sigma = \sigma_b$$

$$= \frac{1}{2}\sqrt{(97.78)^2 + 4(12.22)^2}$$

$$\tau_{\max} = 50.39\, N/mm^2$$

Example 1.15: For the member shown in figure-1.41, determine the stress induced at A and B in the member.

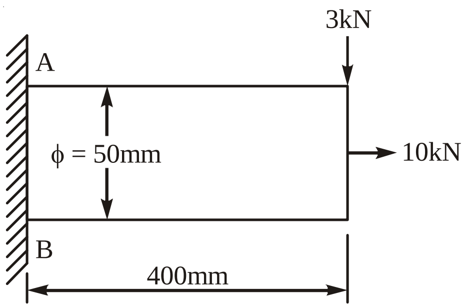

Figure-1.41

Solution:

The axial load 10kN causes tension and tensile stress is induced at critical points A and B. The vertical load causes bending stress at A and B (tension at A and compression at B).

Shear stress is zero at points A and B

Analysis for stress at point A

Therefore, maximum normal stress at A is,

$$\sigma = Tensile\ stress + Bending\ stress$$

$$= \sigma_t + \sigma_b$$

Where, $$\sigma_t = \frac{F}{A}$$

$$= \frac{10\times10^3}{\frac{\pi(50)^2}{4}} = 5.093\, N/mm^2$$

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = 3kN \times 400$

$= 3 \times 10^3 \times 400$ N-mm

$= 12 \times 10^5$ N-mm

$$Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$$

$$= \frac{\pi}{32}(50)^3 = 12271.85 mm^3$$

Therefore, $\sigma_b = \dfrac{12 \times 10^5}{12271.85} = 97.78\, N/mm^2$

$$\sigma = 5.093 + 97.78 = 102.87\, N/mm^2\, (tension)$$

Principal Stresses

Now, principal stress

$$\sigma_1 = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4 \times 0}\right]$$

$$= \sigma_t = 102.87\, N/mm^2\, (tensile)$$

Maximum Shear Stress $\tau_{\max}$

Maximum shear stress $\tau_{\max} = \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$= \frac{1}{2}\sqrt{^2 + 4 \times 0}$$

$$= \frac{\sigma}{2} = \frac{102.87}{2} = 51.43\, N/mm^2$$

Analysis for stress at point B

Maximum normal stress at point $B = \sigma_{tension} - \sigma_b$

$$= 5.093 - 97.78$$

$$= -92.69\, N/mm^2 \,(compressive)$$

Principal Stress

$$\sigma_1 = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\times 0}\right]$$

$$= \frac{1}{2}\left[-92.69 + \sqrt{(-92.69)^2}\right] = 0$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(-92.69)^2 + 0}$$

$$= 46.35\, N/mm^2$$

Example 1.16: A circular shaft of diameter 50mm fixed at one end is subjected to an axial tensile load 10kN and torque of 300N-m. Determine the nature and magnitude of stresses at the critical points.

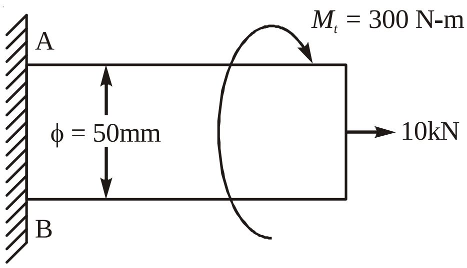

Figure-1.42

Solution:

The tensile load causes tensile stress in the member and torque M_t causes torsional shear stress.

Due to tensile load $F = 10kN$

Direct tensile stress, $\sigma_t = \dfrac{F}{Area\ of\ cross\ section} = \dfrac{F}{\dfrac{\pi d^2}{4}}$

$$= \frac{4F}{\pi d^2} = \frac{4 \times 10 \times 10^3}{\pi (50)^2} = 5.093\, N/mm^2$$

Torque, $M_t = 300$ N-m

$= 300 \times 10^3$ N-mm

Torsional shear stress,

$$\tau = \frac{M_t}{Z_t}$$

$$= \frac{M_t}{\dfrac{\pi d^3}{16}} = \frac{16 M_t}{\pi d^3}$$

$$= \frac{16 \times 300 \times 10^3}{\pi (50)^3} = 12.22\, N/mm^2$$

To find Principal Stresses

Maximum principal stress, $\sigma_1 = \dfrac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

$$= \frac{1}{2}\left[5.093 + \sqrt{(5.093)^2 + 4(12.22)^2}\right]$$

$$= 15.03\, N/mm^2\ (tensile)$$

Minimum principal stress, $\sigma_2 = \dfrac{1}{2}\left[\sigma - \sqrt{\sigma^2 + 4\tau^2}\right]$

$$= \frac{1}{2}\left[5.093 - \sqrt{(5.093)^2 + 4(12.22)^2}\right]$$

$$= -9.94\, N/mm^2\ (compressive)$$

Maximum Shear stress τ_{max}

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(5.093)^2 + 4(12.22)^2}$$

$$\tau_{max} = 12.78\, N/mm^2$$

Example 1.17: A rod of 50mm diameter is subjected to combined bending, axial and torsional loads as shown in figure-1.43. Determine the nature and magnitude of stresses at the critical points.

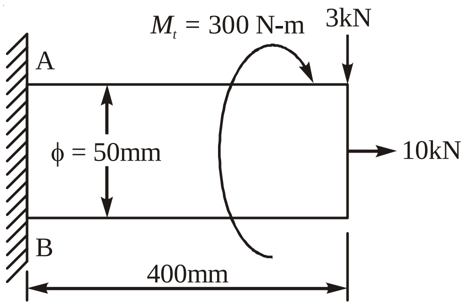

Figure-1.43

Solution:

The critical points in the member are A and B.

Tensile load 10kN causes tensile stress, point load 3kN cause bending stress and the torque M_t causes torsional shear stress τ .

Stress at point A

i) Tensile stress due to 10kN,

$$\sigma_t = \frac{F}{Area\ of\ cross\ section} = \frac{F}{\frac{\pi d^2}{4}}$$

$$= \frac{4F}{\pi d^2} = \frac{4 \times 10 \times 10^3}{\pi (50)^2} = 5.093\, N/mm^2$$

ii) Bending stress due to 3kN is given by

$$\sigma_b = \frac{M_b}{Z_b} = \frac{M_b}{\frac{\pi d^3}{32}}$$

Where, $M_b = 3kN \times 400$

$$= 3 \times 10^3 \times 400 \text{ N-mm}$$

$$Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$$

Therefore, $\sigma_b = \frac{32 M_b}{\pi d^3}$

$$= \frac{32 \times 3 \times 10^3 \times 400}{\pi (50)^3} = 97.8\, N/mm^2$$

Normal stress at A is

$$\sigma = \sigma_t + \sigma_b$$

$$= 5.093 + 97.8 = 102.893\, N/mm^2$$

iii) Torsional shear stress due to M_t is

$$\tau = \frac{M_t}{Z_t} = \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16 M_t}{\pi d^3}$$

$$= \frac{16 \times 300 \times 10^3}{\pi (50)^3} = 12.22\, N/mm^2$$

$$Z_t = \frac{J}{c} = \frac{\pi d^3}{16}$$

Principal stresses

Maximum principal stress,

$$\sigma_1 = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[102.893 + \sqrt{(102.893)^2 + 4(12.22)^2}\right]$$

$$= 104.32\, N/mm^2\ (tensile)$$

Minimum principal stress,

$$\sigma_2 = \frac{1}{2}\left[\sigma - \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[102.893 - \sqrt{(102.893)^2 + 4(12.22)^2}\right]$$

$$= 102.863\, N/mm^2\ (compressive)$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(102.893)^2 + 4(12.22)^2}$$

$$= 52.88\, N/mm^2$$

Stress at point B

Normal stress $\sigma = \sigma_t - \sigma_b$

$$\sigma = 5.093 - 97.8 = -92.71\, N/mm^2$$

Torsional shear stress

$$\tau = \frac{M_t}{Z_t} = \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16 M_t}{\pi d^3}$$

$$\frac{16 \times 300 \times 10^3}{\pi(50)^3} = 12.22\, N/mm^2$$

Section modulus in torsion

$$Z_t = \frac{J}{c} = \frac{\pi d^3}{16}$$

Principal stresses

Maximum principal stress,

$$\sigma_1 = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[(-92.71) + \sqrt{(92.71)^2 + 4(12.22)^2}\right]$$

$$= -1.584\, N/mm^2\, (tensile)$$

Minimum principal stress,

$$\sigma_2 = \frac{1}{2}\left[\sigma - \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$= \frac{1}{2}\left[(-92.71) - \sqrt{(92.71)^2 + 4(12.22)^2}\right]$$

$$= -188.59\, N/mm^2\, (compressive)$$

Maximum shear stress

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(-92.71)^2 + 4(12.22)^2}$$

$$= 47.94\, N/mm^2$$

Example 1.18: For the Z-bracket shown in figure-1.44, determine the maximum shear stress at section 1-1 and 2-2.

Solution:

At section 1-1, $\sigma_x = \sigma_d + \sigma_{b(1-1)}$

Point A on section 1-1 is subjected to compression due to load 50kN and hence,

$$\sigma_d = \frac{F}{A}$$

Where, F is the load on member $= 50 \times 10^3\, N$

A is the area of cross section of member $= 150 \times 60 = 900 mm^2$

$$\sigma_{b\,at1-1} = \frac{M_b}{Z_b}$$

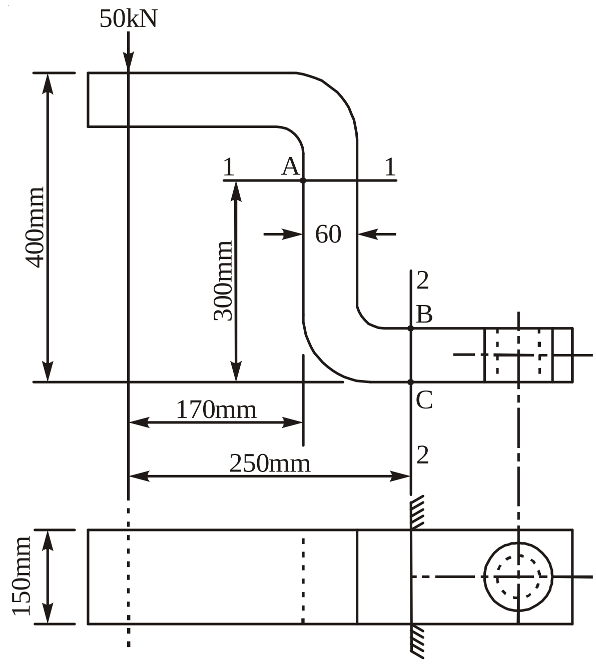

Figure-1.44

Where, $$M_b = F\left(170 + \frac{60}{2}\right)$$

$$= 50 \times 10^3 \left(170 + 30\right)$$

$$= 10000 \times 10^3 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

Where, $$c = \frac{Depth}{2} = \frac{60}{2} = 30mm$$

$$I = \frac{150\left(60\right)^3}{12} mm^4 = 27 \times 10^5 mm^4$$

Therefore, $$\sigma_x = \frac{50 \times 10^3}{9000} - \frac{1000 \times 10^3 \times 30}{27 \times 10^5}$$

$$= -5.56 - 111 - 11$$

$$= -116.67\ N/mm^2$$

Therefore, maximum shear stress

$$\tau_{\max} = \frac{116.67}{2} = 58.34\ N/mm^2$$

For section 2-2 of bracket

$$\sigma_x = \sigma_{b\,at\,2-2},\ where\ \sigma_{b\,at\,2-2} = \frac{M_b}{Z_b}$$

And $M_b = \left(50\times10^3\right)250$ N-mm

$$= 1250\times10^5 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

$$I = \frac{150\times60^3}{12} = 12\times10^5 mm^4$$

$$c = \frac{60}{2} = 30mm$$

$$\sigma_x = \sigma_{b(2-2)} = \frac{125\times10^5\times30}{27\times10^5}$$

$$= 138.89\ N/mm^2$$

Note: ***Point B on section 2-2 is subjected to tension and point C is subjected to compression.***

Therefore, maximum shear stress at section 2-2 is

$$\tau_{\max} = \frac{\sigma_x}{2} = \frac{138.89}{2} = 69.45\ N/mm^2$$

Example 1.19: An electric motor weighing 2kN is provided with an eye bolt as shown in figure-1.45 for the purpose of lifting. Determine the core area of the bolt assuming permissible tensile stress in the bolt as $50\ N/mm^2$.

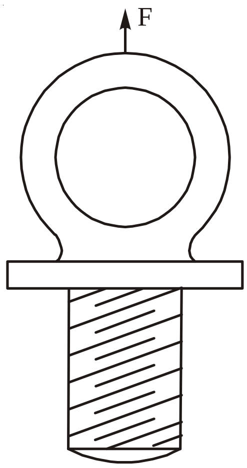

Figure-1.45

Solution:

The load $F = 3kN$ produces tensile stress in the threaded portion given by

$$\sigma_t = \frac{F}{A_c}$$

Where, $\sigma_t = Allowable\ tensile\ stress = 50\ N/mm^2$

$F = Tensile\ load = 3kN = 3\times10^3\ N$

$A_c = Core\ area\ of\ bolt, mm^2$

Therefore, $A_c = \frac{F}{\sigma_t}$

$$= \frac{3\times10^3}{50} = 60mm^2$$

Example 1.20: Determine the shear force required to punch a 20mm diameter hole in a mild steel plate 15mm thick shown in figure-1.46 taking the ultimate shear stress as $400\ N/mm^2$.

Solution:

$Area\ under\ shear = Circumference\ of\ hole \times Thickness\ of\ plate$

$$= \pi dt$$

$$= \pi \times 20\times15 = 942.48mm^2$$

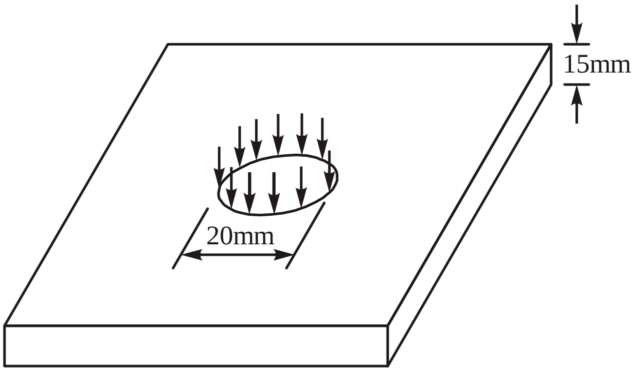

Figure-1.46

$$\tau_u = Ultimate\ shear\ stress = \frac{Shear\ \ force}{Area\ under\ shear}$$

$$= \frac{F}{942.48}$$

Therefore, $F = 942.48 \times \tau_u$

$= 942.48 \times 400$

$= 376692N = 376.992kN$

Example 1.21: A mild steel shaft 100mm in diameter weighing 20kN is supported between two bearings. Determine the length of the bearing required taking permissible bearing pressure = 5MPa. Determine the bearing pressure induced if the bearing is changed to a square bearing.

Solution:

i) $$Bearing\ \ pressure = \frac{Bearing\ \ load}{Projected\ area\ of\ \ bearing}$$

Here, *Bearing pressure = 5MPa*

$$Bearing\ \ load\ \ per\ \ bearing = \frac{20kN}{Number\ \ of\ bearings}$$

$$= \frac{20 \times 10^3 N}{2} = 10 \times 10^3$$

$Projected\ \ area\ \ of\ \ bearing = ld$

$$= l \times 100 mm^2$$

Therefore, $5 = \dfrac{10 \times 10^3}{l \times 100}$

Therefore, *Length of bearing 'l'= 20mm.*

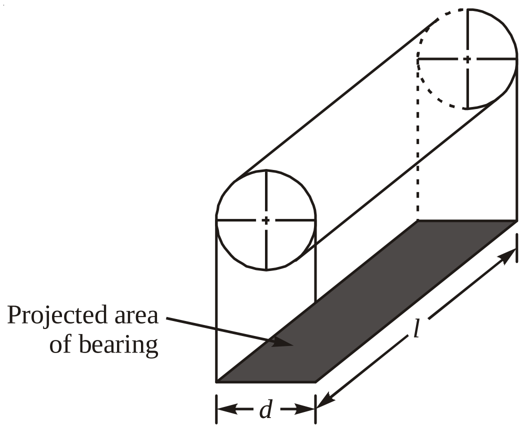

Figure-1.47

ii) For a square bearing, $l = d$

Therefore, $Bearing\ \ pressure = \dfrac{F}{dd}$

$$= \frac{10 \times 10^3}{100 \times 100} = 1\, N/mm^2 = 1 MPa$$

Example 1.22: A link shown in figure-1.48 is subjected to a pull of 5kN. Determine the dimensions of the cross section of the beam if depth of beam h = 3 times width b. The allowable tensile stress for the material of the link is $80\, N/mm^2$.

Solution:

The critical cross section A-A of beam is subjected to tensile stress σ_t due to F and bending stress due to ecentricity of load.

Maximum tensile stress occurs at the upper portion 1-1 and compressive stress at the bottom portion.

Therefore, *The maximum tensile stress* $\sigma_{\max} = \sigma_t + \sigma_b$

Where, $\sigma_t = \dfrac{F}{Area\ of\ \ cross\ section}$

$$= \frac{5\times10^3}{bh}$$

$$= \frac{5\times10^3}{b\times3b} = \frac{1666.67}{b^2}$$

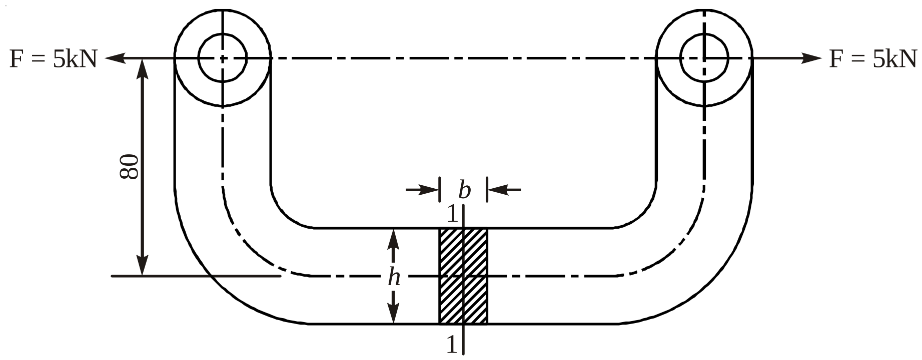

Figure-1.48

Bending stress, $\sigma_b = \dfrac{M_b}{Z_b}$

Where, $Z_b = \dfrac{I}{c}$

$$\sigma_b = \frac{M_b}{\frac{I}{c}} = \frac{M_b c}{I}$$

$$M_b = 5\times10^3\times80$$

$$= 400\times10^3 \text{ N-mm}$$

$$I = \frac{bh^3}{12} = \frac{b(3b)^3}{12}$$

$$= \frac{27}{12}b^4 = \frac{9}{4}b^4;$$

$$c = \frac{h}{2}$$

$$= \frac{3b}{2} = 1.5b$$

$$\sigma_b = \frac{400 \times 10^3 \times 1.5b}{\frac{9}{4b^4}} = \frac{266666.67}{b^3}$$

Therefore, $$80 = \frac{1666.67}{b^2} + \frac{266666.67}{b^3}$$

$$80b^3 = 1666.67b + 266666.67$$

$$b^3 = 20.833b + 3333.33$$

By trial and error , width of link b = 15mm.
Therefore, The dimension of cross section of link are

$$Width \quad b = 15mm$$

$$Depth \quad h = 3b = 3 \times 15 = 45mm$$

Example 1.23: A beam of rectangular cross section with depth equal to twice width and supported between bearings 2 metres apart has a uniformly distributed load of $10\,kN/m$. ***Taking 45C8 as the material and beam and factor of safety 4, determine the dimensions of the cross section of beam.***

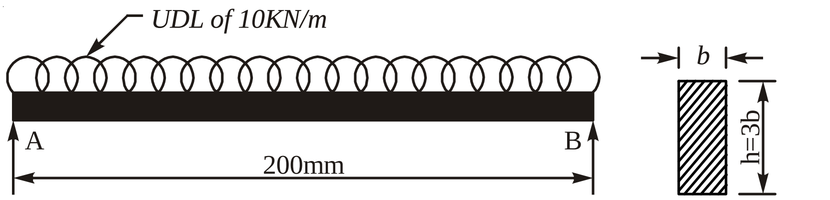

Figure-1.49

Solution:

The beam AB is subjected to bending due to uniformly distributed load. The lower surface is subjected to bending stress (tensile) and the upper surface to bending stress (compressive).

To calculate the maximum bending stress occurring at the centre of beam

The maximum bending moment $M_{b\max} = \dfrac{Wl^2}{8}$

Where, $W = Intensity\ of\ \ loading = 10\,kN/m = 10 \times 10^3$ N-m, Length of beam $l = 2m.$

Therefore, $$M_{b\max} = \frac{\left(10\times10^3\right)2^2}{8}$$

$$= 5\times10^3 \text{ N-m} = 5\times10^6 \text{ N-mm}$$

Now, Bending stress $\sigma_b = \dfrac{M_{b\max}}{Z_b}$

Where, $$Z_b = \frac{I}{c}$$

$$= \frac{\left(\frac{b^3}{12}\right)}{\left(\frac{h}{2}\right)} = \frac{bh^2}{6} = \frac{b\left(3b\right)^2}{6} = 1.5b^3$$

For 45C8 steel, from Table (1-21),

$$\sigma_{ut} = 618MPa$$

Assuming, $\sigma_{yt} = 0.6\sigma_{ut},$

$$\sigma_{yt} = 0.6\times618 = 370.8MPa$$

Design stress $= \dfrac{\sigma_{yt}}{f.o.s} = \dfrac{370.8}{4} = 92.7MPa$

From, $\sigma_b = \dfrac{M_{b\max}}{Z_b},$

$$92.7 = \frac{5\times10^6}{1.5b^3}$$

Therefore *Use width* $b = 33mm$ *and*
depth $h = 3b = 3\times33 = 99mm.$

Example 1.24: The propeller shaft of a marine vessel is subjected to an axial load of 4kN, torque of 6kN-m and bending moment of 3kN-m at the critical section. Determine the diameter of the shaft taking the design stress for the material of the shaft as 100MPa.

Solution:

At the critical section,

i) ***Direct tensile stress due to axial load 4kN is***

$$\sigma_t = \frac{F}{A} = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

Where, *d is the shaft diameter.*

$$\sigma_t = \frac{4\left(4\times10^3\right)}{\pi d^2} = \frac{5092.95}{d^2}(tensile)$$

ii) ***Bending stress due to bending moment 3kN-m is***

$$\sigma_b = \frac{M_b}{Z_b} = \frac{M_b}{\frac{\pi d^3}{32}} = \frac{32M_b}{\pi d^3}$$

Where, $M_b = 3$ kN-m

$= 3\times10^6$ N-mm

Therefore, $\sigma_b = \frac{32\times10^6}{\pi d^3}$

$$= \frac{30.56\times10^6}{d^3}(tensile)$$

iii) ***Shear stress due to torque 6 kN-m is***

$$\tau = \frac{M_t}{Z_t} = \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16M}{\pi d^3}$$

$M_t = 6$ kN-m

$= 6\times10^6$ N-mm

$$\tau = \frac{16\times6\times10^6}{\pi d^3}$$

$$\tau = \frac{30.56 \times 10^6}{d^3}$$

Now Maximum principal stress,

$$\sigma_1 = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

Solve *'d'* by trial and error method.

Example 1.25: A beam of square section $50mm \times 50mm$ ***and of length 1 metre, supported between bearings supports a load of 5kN at its centre as shown in figure-1.50. Determine the bending stress induced in the beam.***

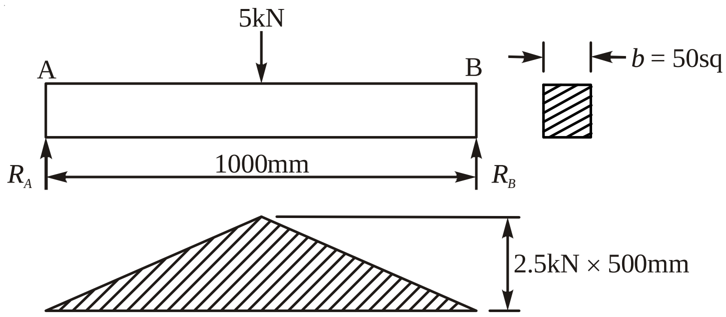

Figure-1.50

Solution:

Reactions at bearings A and B are equal and are given by

$$R_A = R_B = \frac{5}{2} kN = 2.5 \times 10^3 N$$

Maximum bending moment at

$$C = R_A \frac{1000}{2}$$

$$= \left(2.5 \times 10^3\right) \times 500$$

$$= 12.5 \times 10^5 \text{ N-mm}$$

Now bending stress, $\sigma_b = \dfrac{M_b}{Z_b}$

Where, $Z_b = Section\ modulus = \dfrac{I}{c}$

$$= \frac{\left(\frac{bh^3}{12}\right)}{\frac{h}{2}} = \frac{bh^2}{6} = \frac{b \times b^2}{6} = \frac{b^3}{6}$$

$$= \frac{(50)^3}{6} = 20833.33\, mm^3$$

Therefore, Bending stress induced in the beam,

$$\sigma_b = \frac{12.5 \times 10^5}{20833.33} = 60\ N/mm^2$$

Note: Bending stress in tensile at the bottom surface of beam and compressive at the top surface.

Example 1.26: A rivet of diameter d is subjected to (i) single shear and (ii) double shear by a load 2kN as shown in figure-1.51. Determine the diameter of the rivet required taking the permissible shear stress = 50MPa.

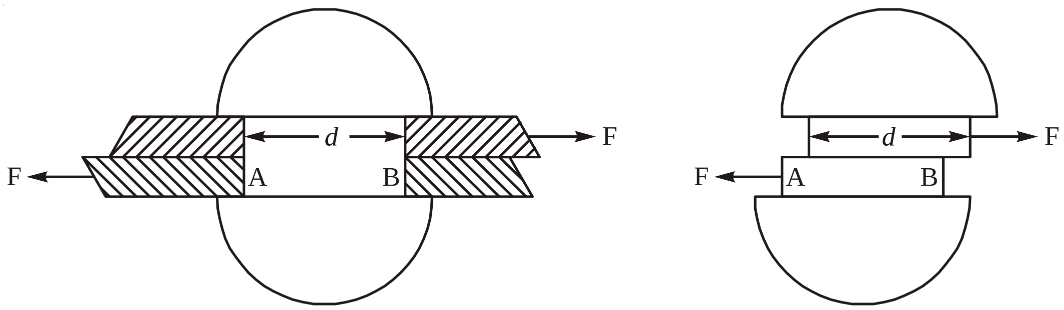

Figure-1.51(a).

Solution:

Figure 1.51(a) shows a rivet in a lap joint (riveted joint) resting load F.

The area resting load $= \frac{\pi d^2}{4}$

Therefore, Shear stress (due to single shear)

$$= \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}; \qquad F = 2kN = 2 \times 10^3 N$$

$$50 = \frac{4 \times 2 \times 10^3}{\pi d^2}$$

Therefore, $\quad Diameter\ of\ rivet = d = 7.135mm$

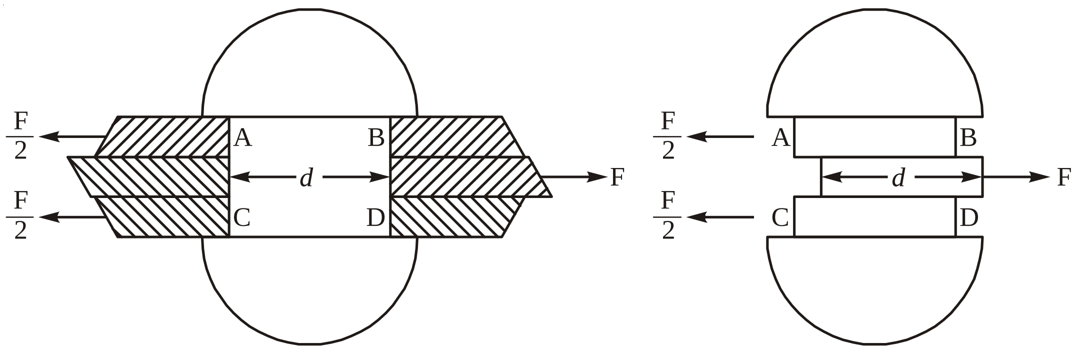

Figure-1.51(b)

The are resting load $\quad = 2\dfrac{\pi d^2}{4}$

Therefore, Shear stress (due to double shear)

$$= \frac{F}{2\left(\dfrac{\pi d^2}{4}\right)} = \frac{2F}{\pi d^2}$$

Shear stress τ,

$$50 = \frac{2 \times 2 \times 10^3}{\pi d^2}$$

Therefore, Diameter of the rivet required, d = 5.05mm.

Example 1.27: A steel shaft shown in figure-1.52 is subjected to torque of 25kN-m. Determine diameter of the shaft required taking permissible shear stress in the shaft as 60MPa.

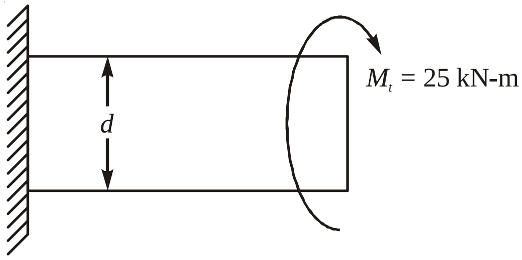

Figure-1.52

Solution:

Due to torque M_t, the torsional shear stress τ induced in the shaft is

$$\tau = \frac{M_t}{Z_t}$$

Where, $M_t = 25$ kN-m

$= 25\times10^3$ N-m $= 25\times10^6$ N-mm

$$Z_t = \frac{J}{c} = \frac{\pi d^3}{16}$$

$$\tau = 60\,N/mm^2$$

i.e., $$60 = \frac{25\times10^6}{\left(\frac{\pi d^3}{16}\right)}$$

Therefore, Diameter of the shaft required, *d = 128.5mm.*

Example 1.28: The frame of a hydraulic press consisting of two identical steel plates shown in figure-1.53. The maximum force P acting on the frame is 20kN. These plates are made up of 45C8 steel with a yield tensile strength of $380\,N/mm^2$. The factor of safety is 2.5. Determine the plate thickness. (VTU, January/February 2005)

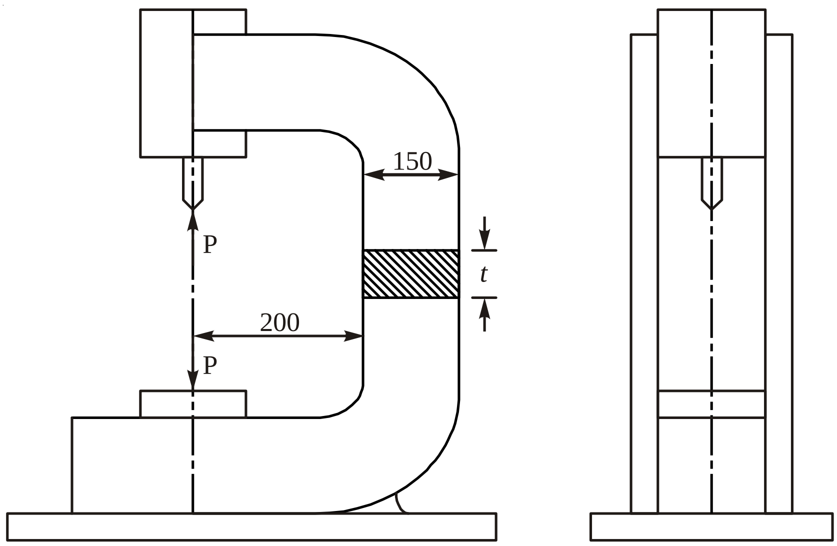

Figure-1.53

Solution:

The cross section of steel plates is subjected to combined bending and tensile load.

Therefore, $\sigma_t = \sigma_d + \sigma_b$ ... (1)

Where, $\sigma_t = \dfrac{\sigma_{yt}}{f.o.s}$

$$= \frac{380}{2.5} = 152\, N/mm^2$$

$$\sigma_d = \frac{P}{2A}$$

$$= \frac{20kN}{2(150 \times t)} = \frac{20 \times 10^3}{2(150t)} = \frac{66.67}{t}$$

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = Pl$

$$= p\left(200 + \frac{150}{2}\right) = 20 \times 10^3 (200 + 75)$$

$$= 5500000 \text{ N-mm}$$

$$Z_b = Section\ modulus = \frac{\frac{t(150)^3}{12}}{\frac{150}{2}} = 3750t$$

Where, *t = thickness of plate; mm*

Therefore, $\sigma_b = \dfrac{5500000}{3750t} = \dfrac{1466.67}{t}$

Substituting the values in equation (1),

$$152 = \frac{66.67}{t} + \frac{1466.67}{t}$$

Therefore, *t, the thickness of plate = 10.087mm.*

Example 1.29: A stepped shaft is subjected to a transverse load of 8kN as shown in figure-1.54. The shaft is made of steel with ultimate tensile strength of 400MPa. Determine the diameter of shaft based on a factor of safety of 2. (VTU, July 2007)

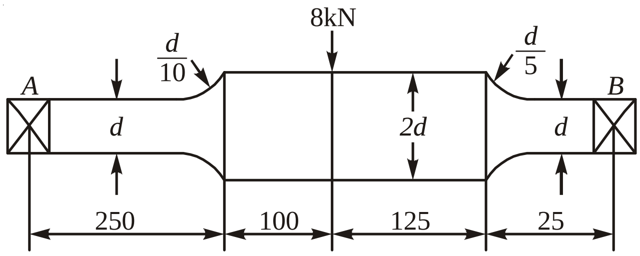

Figure-1.54

Solution:

To find reactions R_A and R_B at bearings A and B due to load 8kN

$$R_A + R_B = 8kN \quad \text{... (1)}$$

Taking moments about bearing *A*,

$$R_B \times 550 = 8 \times 350$$

$$R_B = 5.0909kN$$

Therefore, $R_A = 2.9091kN$

Case (i) Analysis of member at the change of cross section

Analysis at section A-A (considering left end bearing)

The member is subjected to bending due to R_A

Therefore, Bending moment $M_b = R_A \times 250$

$$= \left(2.9091 \times 10^3\right) 250$$

$$= 727275 \text{ N-mm}$$

For the portion of shaft to the left of load 8kN,

$$\frac{r}{d} = \frac{\frac{d}{10}}{d} = 0.1$$

$$\frac{D}{d} = \frac{2d}{d} = 2$$

From figure (4.21A MDH), $K_\sigma = 1.74$

But, $K_\sigma = \frac{\sigma_{b\max}}{\sigma_{bnom}}$

Where, $\sigma_{b\max} = \frac{\sigma_{ut}}{f.o.s} = \frac{400}{2} = 200MPa$

Therefore, $\sigma_{bnom} = \frac{200}{1.74} = 114.94\,N/mm^2$

Therefore, $\sigma_{bnom} = \frac{\pi d^3}{32} = 114.94$

Therefore $d = 10.54mm.$

Example 1.30: The shaft of a motor is supported at two points which are 800mm apart. The armature of motor can be considered as a uniformly distributed load of 15 N/mm, centrally spread over a length of 500mm. Selecting a suitable material and choosing an appropriate value for the factor of safety, determine the diameter of the motor shaft.

(VTU, July 2006)

Solution:

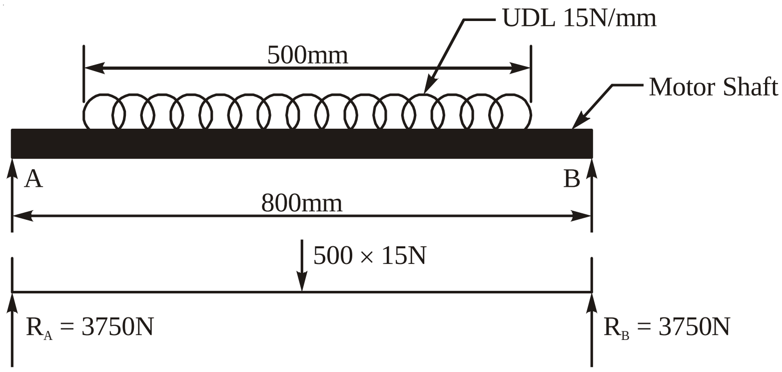

Figure-1.55

The shaft is subjected to bending moment M_b

Therefore, Bending stress induced in the shaft is given by

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = R_A \times 400$

$= 3750 \times 400 = 15 \times 10^5$ N-mm

$$Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$$

Assuming C40 (i.e. 40C8) as the material of shaft,

$$\sigma_{ut} = 600MPa$$

Assuming $\sigma_y = 0.6 \times \sigma_{ut}$

$$\sigma_y = 0.6 \times 600 = 360MPa$$

Taking factor of safety = 3,

$$\sigma_b = \frac{\sigma_y}{f.o.s} = \frac{360}{3} = 120MPa$$

Substituting in $\sigma_b = \frac{M_b}{Z_b}$

$$120 = \frac{15 \times 10^5 \times 32}{\pi d^3}$$

Therefore, $d = 50.31mm$.

Example 1.31: A steel member is loaded as shown in figure-1.56. Determine the magnitude of

i) Maximum tensile stress.

ii) Maximum shear stress. ***(VTU, July 2007)***

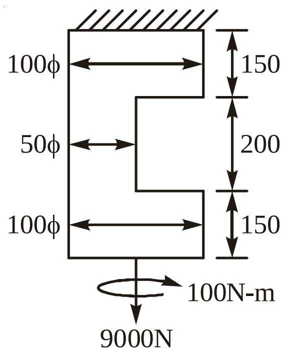

Figure-1.56

Solution:

The steel member is subjected to combined bending, axial and torsional loads.

i) Tensile stress due to axial load 9000N.

ii) Bending stress due to load 9000N due to eccentricity *e*.

iii) Torsional shear stress due to torque 100 N-m.

i) To find maximum normal stress

Maximum normal stress $= \sigma_d + \sigma_b$

Where, $\sigma_d = \dfrac{P}{A_{\min}}$

$$= \frac{9000}{\frac{\pi}{4}(50)^2} = 4.584\, N/mm^2$$

Bending stress, $\sigma_b = \dfrac{M_b}{Z_b}$

Where, $M_b = P\left(\dfrac{d}{2}\right)$

$$= 9000 \times 25 = 225000 \text{ N-mm}$$

$$Z_b = Section\ modulus = \frac{\pi d^3}{32}$$

$$= \frac{\pi}{32}(50)^3 = 12271.85\, mm^3$$

Therefore, $\sigma_b = \dfrac{225000}{12271.85} = 18.335\, N/mm^2$

Therefore, Maximum normal stress,

$$= 4.584 + 18.335$$

$$= 22.919\, N/mm^2$$

ii) To find maximum shear stress $\tau_{\max}$

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

Where, $\sigma = \sigma_b = 18.335\, N/mm^2$

$$\tau = \frac{M_t}{Z_t}$$

Where, $M_t = 100 \text{ N-m} = 100 \times 10^3 = 10^5 \text{ N-mm}$

$$Z_t = \frac{\pi d^3}{16}$$

$$= \frac{\pi}{16}(50)^3 = 24543.69\,mm^3$$

Therefore, $\tau = \dfrac{10^5}{24543.69} = 4.074\,N/mm^2$

Therefore, $\tau_{\max} = \dfrac{1}{2}\sqrt{(22.919)^2 + 4(4.074)^2}$

$$\tau_{\max} = 12.162$$

Example 1.32: A hot rolled bar has an yield strength of 390Mpa. Compute the factor of safety using the following theories of failure.

i) ***Maximum normal stress theory.***

ii) ***Maximum shear stress theory and***

iii) ***Distortion energy theory for the following state of stress.***

a) $\sigma_1 = 225MPa,$ $\sigma_2 = 225MPa,$ $\sigma_3 = 0$

b) $\sigma_1 = 225MPa,$ $\sigma_2 = 120MPa,$ $\sigma_3 = 0$

c) $\sigma_1 = 225MPa,$ $\sigma_2 = 0,$ $\sigma_3 = -120MPa$

(VTU, December 2006 / January 2007)

Solution:

i) ***Maximum normal stress theory.***

a) $\sigma_{\max} = \sigma_1 = \dfrac{\sigma_{yt}}{f.o.s};$ $\sigma_1 = 225$

i.e., $225 = \dfrac{\sigma_{yt}}{f.o.s} = \dfrac{390}{f.o.s}$

Therefore, $f.o.s = \dfrac{390}{225} = 1.7333$

b) $\sigma_{\max} = \sigma_1 = \dfrac{\sigma_{yt}}{f.o.s};$ $\sigma_1 = 225$

i.e., $$225 = \frac{\sigma_{yt}}{f.o.s} = \frac{390}{f.o.s}$$

Therefore, $$f.o.s = \frac{390}{225} = 1.7333$$

c) $$\sigma_{max} = \sigma_1 = \frac{\sigma_{yt}}{f.o.s}; \qquad \sigma_1 = 225$$

i.e., $$225 = \frac{\sigma_{yt}}{f.o.s} = \frac{390}{f.o.s}$$

Therefore, $$f.o.s = \frac{390}{225} = 1.7333$$

ii) Maximum shear stress theory.

a) $$\sigma_1 - \sigma_3 = \frac{\sigma_{yt}}{f.o.s}$$

Therefore, $$f.o.s = \frac{\sigma_{yt}}{\sigma_1 - \sigma_3} = \frac{390}{225 - 0} = 1.7333$$

b) $$\sigma_1 - \sigma_3 = \frac{\sigma_{yt}}{f.o.s}$$

Therefore, $$f.o.s = \frac{\sigma_{yt}}{\sigma_1 - \sigma_3} = \frac{390}{225 - 0} = 1.7333$$

c) $$\sigma_1 - \sigma_3 = \frac{\sigma_{yt}}{f.o.s}$$

Therefore, $$f.o.s = \frac{\sigma_{yt}}{\sigma_1 - \sigma_3} = \frac{390}{225 - (-120)} = 1.13$$

iii) Distortion energy theory

a) $$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = \frac{\sigma_{yt}}{f.o.s}$$

$$= \sqrt{\frac{(225-225)^2 + (225-0)^2 + (0-225)^2}{2}} = 225$$

Therefore, $225 = \dfrac{\sigma_{yt}}{f.o.s}$

i.e., $f.o.s = \dfrac{390}{225} = 1.7333$

b) $\sigma' = \sqrt{\dfrac{(225-120)^2 + (120-0)^2 + (0-225)^2}{2}} = \dfrac{\sigma_{yt}}{f.o.s}$

$= 195$

i.e., $f.o.s = \dfrac{390}{\sigma'} = \dfrac{390}{195} = 2$

c) $\sigma' = \sqrt{\dfrac{(225-0)^2 + (0+120)^2 + (-120-225)^2}{2}} = \dfrac{\sigma_{yt}}{f.o.s}$

$= 303.36$

Therefore, $303.36 = \dfrac{\sigma_{yt}}{f.o.s}$

i.e., $f.o.s == \dfrac{390}{303.36} = 1.2856$

Example 1.33: The state of stresses at a point in a body are $\sigma_x = 81MPa,\ \sigma_y = 21MPa$ ***and*** $\tau_{xy} = 84MPa.$ ***The yield stress of material is 280MPa. Find the factor of safety by***

i) Maximum shear stress.

ii) Distortion energy theory. ***(VTU, June / July 2008)***

Solution:

Maximum principal stress,

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... E(2-34)}$$

$$= \left(\frac{81+21}{2}\right) + \sqrt{\left(\frac{81-21}{2}\right)^2 + (84)^2}$$

$$= 140.196\, N/mm^2$$

Maximum principal stress,

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{81+21}{2}\right) - \sqrt{\left(\frac{81-21}{2}\right)^2 + (84)^2}$$

$$= -38.196\, N/mm^2$$

i) ***According to maximum shear stress theory, the maximum stress induced in member is***

$$\tau_{\max} = Largest\ of\ \frac{1}{2}\left|(\sigma_1 - \sigma_2)\right|,\ \frac{1}{2}\left|(\sigma_1)\right|,\ \frac{1}{2}\left|(\sigma_2)\right|$$

$$= \frac{1}{2}\left|(140.196 + 38.196)\right|,\ \frac{1}{2}\left|(140.196)\right|,\ \frac{1}{2}\left|(-38.196)\right|$$

$$= 89.196,\ 70.093,\ 19.093$$

Therefore, $\tau_{\max} = 89.193\ (largest\ value)$

But, $\tau_{\max} = \dfrac{\sigma_{yt}}{2 \times f.o.s}$

Therefore, $f.o.s = \dfrac{280}{2 \times 89.16} = 1.5696$

ii) ***Distortion energy theory***

$$\frac{\sigma_{yt}}{f.o.s} = \sqrt{(\sigma_1)^2 + (\sigma_2)^2 - \sigma_1\sigma_2}$$

i.e., $\dfrac{280}{f.o.s} = \sqrt{(140.196)^2 + (-38.196)^2 - (140.196 \times -38.196)}$

$$= 162.69$$

Therefore, $f.o.s = \frac{280}{162.69} = 1.7211$

Example 1.34: Determine the extreme fibre stresses for the cross section X-X of the C-clamp loaded as shown in figure-1.57. (VTU, July 2006)

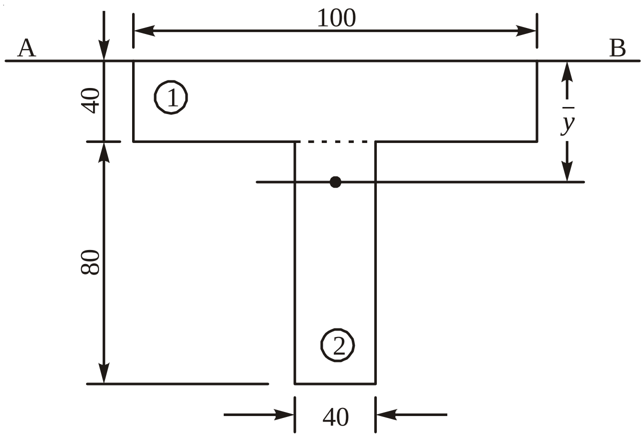

Figure-1.57

Solution:

Tensile stress in the C-clamp is given by

$$\sigma_t = \sigma_d + \sigma_b \quad \text{... (1)}$$

To find σ_t

Assuming 40C8 as the material of clamp,

$$\sigma_{ut} = 600\ N/mm^2 \quad \text{... T(1.21)}$$

Taking $\sigma_y = 0.6\sigma_{ut}$,

$$\sigma_y = 0.6 \times 600 = 360\ N/mm^2$$

Using factor of safety = 3,

$$\sigma_t = \frac{\sigma_y}{f.o.s} = \frac{360}{3} = 120\ N/mm^2$$

To find σ_d

$$\sigma_d = \frac{P}{A}$$

Where, $P = 100kN = 100 \times 10^3 N$

A = Area of cross section of critical section of clamp.

$$= (100 \times 40) + (80 \times 40) = 7200 \ mm^2$$

Therefore, $\sigma_d = \dfrac{100 \times 10^3}{7200} = 13.89 \ N/mm^2$

To find σ_b

$$\sigma_b = Bending \ stress = \frac{M_b}{Z_b}$$

$$M_b = pl$$

$$= p\left(200 + \overline{y}\right)$$

$$= \left(100 \times 10^3\right)\left(200 + \overline{y}\right)$$

To find $\overline{y}$

Taking line AB as reference,

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Where, $a_1 = 100 \times 40 = 4000 \ mm^2$

$$a_2 = 80 \times 40 = 3200 \ mm^2$$

$$y_1 = 20 mm$$

$$y_2 = 40 + \frac{80}{2} = 80 mm$$

Therefore, $\overline{y} = \dfrac{(4000)20 + (3200)80}{7200}$

$$= 46.67 mm$$

Therefore, $M_b = 100 \times 10^3 (200 + 46.67)$

$$= 24667000 \text{ N-mm}$$

To find Z_b, the section modulus

$$Z_b = \frac{I}{c}$$

Where, $$I = \frac{100\times 40^3}{12} + a_1\left(\overline{y} - y_1\right)^2 + \frac{40\times 80^3}{12} + a_2\left(\overline{y} - y_2\right)^2$$

$$= \frac{400\times 40^3}{12} + 4000\left(46.67 - 20\right)^2 + \frac{40\times 80^3}{12} + 3200\left(46.67 - 80\right)^2$$

$$= 864.10^4\ mm^3$$

$$c = \overline{y} = 46.67mm$$

Therefore, $$Z_b = \frac{864\times 10^4}{46.67} = 18.513\times 10^4\ mm^3$$

Hence $$\sigma_b = \frac{246670000}{18.513\times 10^4} = 133.24\,N/mm^2$$

Stresses at extreme fibres

Stress at the inner fibre $\sigma_i = \sigma_d + \sigma_b$

$$= 13.89 + 133.24 = 147.13\,N/mm^2\left(tensile\right)$$

Stress at the outer fibre $\sigma_0 = \sigma_d - \sigma_b$

$$= 13.89 - 133.24 = -119.35\,N/mm^2\left(compressive\right)$$

Example 1.35: A steel member is loaded as shown in figure-1.58. Determine the magnitude of

i) The maximum normal stress and

ii) The maximum shear stress. ***(VTU, July 2007)***

Solution:

The member is subjected to

i) Tensile stress due to axial load 9000N.

ii) Bending stress due to 9000N load due to eccentricity $e = 25mm$ and

iii) Torsional shear stress due to torque 100 N-m.

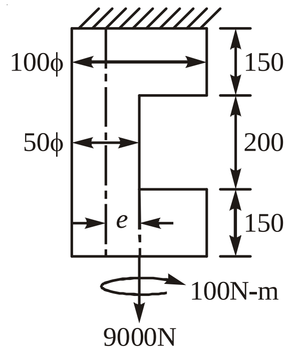

Figure-1.58

i) ***Tensile stress σ_t due to 9000N is given by,***

$$\sigma_t = \frac{P}{A} = \frac{9000}{\frac{\pi}{4}(50)^2} = 4.584\, N/mm^2$$

ii) ***Bending stress σ_b due to load acting at a distance of 25mm from the centre line of 50mm diameter is***

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = Pe = 9000 \times 25 = 225000$ N-mm

$$Z_b = \frac{\pi d^3}{32} = \frac{\pi}{32}(50)^3 = 12271.85\ mm^3$$

Therefore, $\sigma_b = \dfrac{225000}{12271.85} = 18.34\, N/mm^2$

iii) ***Torsional shear stress τ due to torque $M_t = 100$ N-m $= 100 \times 10^3$ N-mm is given by***

$$\tau = \frac{M_t}{Z_t}$$

$$= \frac{100 \times 10^3}{\frac{\pi (50)^3}{16}} = 4.07\, N/mm^2$$

Therefore Maximum normal stress is given by,

$$\sigma = \sigma_t + \sigma_b$$

$$= 4.584 + 18.34 = 22.924\ N/mm^2$$

Maximum shear stress is given by,

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(22.924)^2 + 4(4.07)^2}$$

$$= 12.163\ N/mm^2$$

Example 1.36: A 50mm diameter steel rod supports a 9kN load and in addition is subjected to torsional moment of 100N-m. Determine the maximum tensile and the maximum shear stresses. ***(VTU, January/February 2004)***

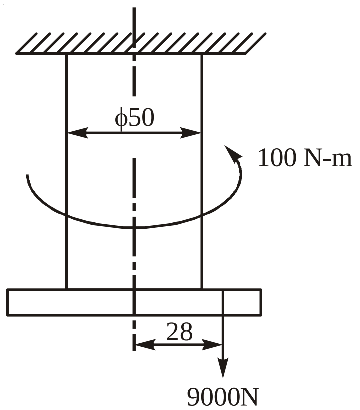

Figure-1.59

Solution:

Point A in the rod is subjected to combined bending, axial and torsional shear load.

The tensile stress is given by,

$$\sigma_t = \frac{P}{A}$$

$$= \frac{9000}{\frac{\pi}{4}(50)^2} = 4.5837\ N/mm^2$$

Bending stress due to 9000N is given by,

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = 9000 \times 28 = 252000$ N-mm

$$Z_b = \frac{\pi d^3}{32} = \frac{\pi}{32}\left(50^3\right) = 12271.85\,mm^3$$

Therefore, $\sigma_b = \dfrac{252000}{12271.85} = 20.53\,N/mm^2$

Torsional shear stress τ ***due to torque*** $M_t = 100$ N-m ***is***

$$\tau = \frac{M_t}{Z_t}$$

$$= \frac{16 M_t}{\pi d^3} = \frac{16 \times 100 \times 10^3}{\pi (50)^3} = 4.074\,N/mm^2$$

Maximum tensile stress,

$$\sigma_{\max} = \sigma_{tensile} = \sigma_t + \sigma_b$$

$$= 4.5837 + 20.53$$

$$= 25.1137\,N/mm^2$$

Maximum shear stress,

$$\tau_{\max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(25.1137)^2 + 4(4.074)^2}$$

$$= 13.2\,N/mm^2$$

Example 1.37: A C-frame of rectangular cross section shown in figure-1.60 is subjected to a load P = 10kN. Determine the dimensions of cross section of beam such that a maximum stress in the frame is not to exceed $120\,N/mm^2$.

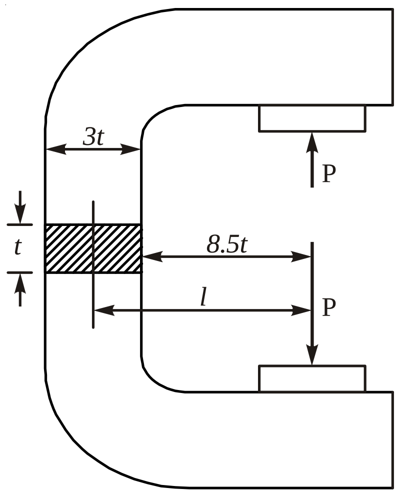

Figure-1.60

Solution:

The maximum stress in the frame due to load *'P'* is

$$\sigma_{max} = \sigma_d + \sigma_b \qquad \text{... (i)}$$

Where, $\sigma_{max} = 120\, N/mm^2$

$$\sigma_d = \frac{P}{A} = \frac{10\times10^3}{3t\times t} = \frac{10^4}{3t^2}$$

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = p\times l$

$$= \left(10\times10^3\right)\left(8.5t + 1.5t\right)$$

$$= 10^5 t \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

$$= \frac{\dfrac{t\left(3t\right)^3}{12}}{\dfrac{1.5t}{1}} = 1.5t^3$$

Substituting in equation (i) yields,

$$120 = \frac{10^4}{3t^2} + \frac{10^5}{1.5t^2}$$

$$= 10^4\left[\frac{1}{3t^2} + \frac{10}{1.5t^2}\right]$$

$$= \frac{10^4}{t^2}\left[\frac{1}{3} + \frac{10}{1.5}\right] = 120$$

Therefore, $t = 24.15\,mm$

Therefore the dimensions of cross section of frame are

Thickness, t = 24.15mm

$Depth = 3t = 72.45mm$

Example 1.38: The frame of a punch press having rectangular section with depth equal to twice width shown in figure-1.61 is to sustain a load of 2kN. Determine the dimensions of cross section by limiting the maximum stress in the frame to 120MPa.

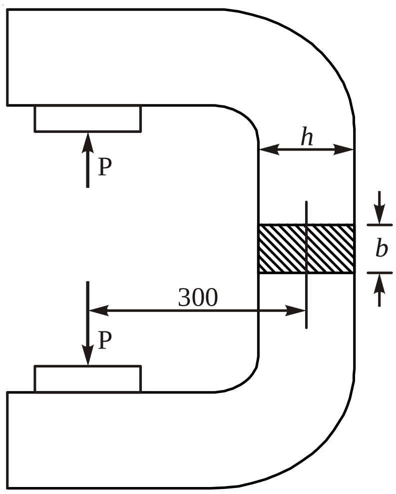

Figure-1.61

Solution:

Maximum stress induced in the frame due to load *'P'* at the inner fibre is given by

$$\sigma = \sigma_d + \sigma_b \quad \text{... (i)}$$

Where, $\sigma_d = Direct\ tensile\ stress\ in\ the\ cross\ section$

$$= \frac{P}{A};\ \ where,\ P\ is\ load = 2kN - 2000N$$

$$A = Area\ of\ \ cross\ section\ of\ \ member$$

$$= b \times h = b \times 2b = 2b^2$$

Therefore, $\sigma_b = \dfrac{2000}{2b^2} = \dfrac{1000}{b^2}$

$$\sigma_b = Bending\ \ stress = \frac{M_b}{Z_b}$$

Where, $M_b = Pl$

$$= 2000 \times 300 = 6 \times 10^5 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

For rectangular section,

$$I = \frac{bh^3}{12}\ \ and\ \ c = \frac{h}{2}$$

Therefore, $Z_b = \dfrac{bh^2}{6}$

$$= \frac{b}{6}\left(2b\right)^2 = \frac{4b^3}{6} = \frac{2}{3}b^3$$

Therefore, $\sigma_b = \dfrac{6 \times 10^6}{2b^3} \times 3 = \dfrac{9 \times 10^5}{b^3}$

$$\sigma = 120MPa\left(given\right)$$

Substituting in equation (i) yields,

$$120 = \frac{1000}{b^2} + \frac{9 \times 10^5}{b^3}$$

i.e., $1 = \dfrac{8.33}{b^2} + \dfrac{7500}{b^3}$

$$b^3 - 8.33b - 7500 = 0$$

By trial and error, b = 19.72mm and h = 2b = 39.44mm.

Therefore, *Adopt b = 20mm and h = 40mm.*

Example 1.39: The frame of a machine member with symmetrical I section shown in figure-1.62 is subjected to a load P. Determine the value of P such that the maximum stress in the frame is not to exceed 100MPa.

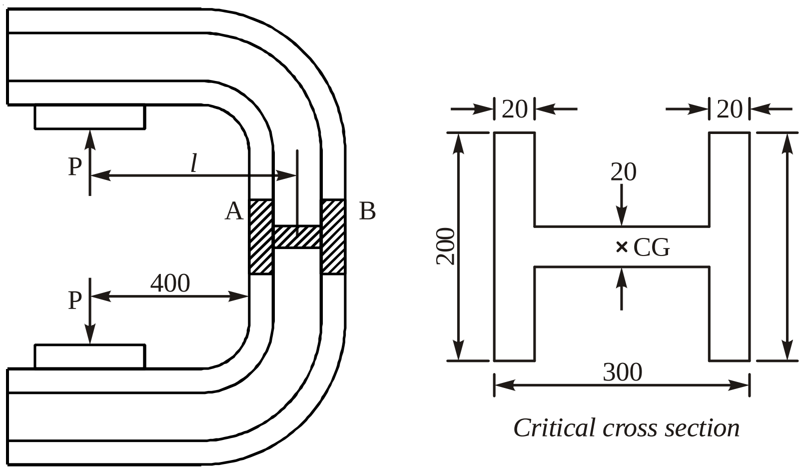

Figure-1.62

Solution:

The maximum stress induced in the member is given by

$$\sigma = \sigma_{\max} = \sigma_d + \sigma_b \quad \text{... (1)}$$

Where, $\sigma_{\max} = 100MPa\left(given\right)$

$$\sigma_d = \frac{Load}{Are\ of\ \ cross\ section} = \frac{P}{A}$$

Where, $P = Load.$

A = Area of cross section of beam

$$= 2\left(200 \times 20\right) + \left(260 \times 20\right) = 13200\, mm^2$$

Therefore, $\sigma_d = \dfrac{P}{13200} N/mm^2$

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = Pl$

$$= P\left(400 + 150\right) \text{N-mm}$$

$$Z_b = \frac{I}{c}$$

Where, $I = Moment\ of\ inertia\ of\ section\ about, G$

$$= 2\left[\frac{200\times 20^3}{12} + (200\times 20)(150-10^2)\right] + \frac{20\times 260^3}{12}$$

$$= 78533333.33\times + 29293333.33$$

$$= 18636\times 10^4\ mm^3$$

$$c = \frac{300}{2} = 150mm$$

Therefore, $\sigma_b = \frac{P(400+150)150}{(18636\times 10^4)}$

Substituting in equation (i) yields,

$$100 = \frac{P}{13200} + \frac{(P\times 400+150)\times 150}{(18636\times 10^4)}$$

$$= P\left[\frac{1}{13200} + 4.427\times 10^{-4}\right]$$

Therefore, $P = 192879.81N = 192.879kN$

Example 1.40: A stationary stepped shaft 1 metre long supported between bearings carries a point load of 3kN at the centre of beam. Determine the dimension of shaft (i.e. D, d and r) such that the maximum stress in the shaft is not to exceed $100\ N/mm^2$.

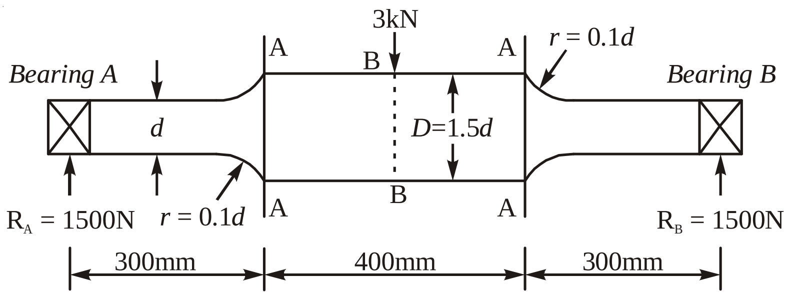

Figure-1.63

Solution:

Because of symmetry, Reactions at bearings A and B are equal i.e., $R_A = R_b = \frac{3kN}{2} = \frac{3 \times 1000}{2} = 1500N$

At the change of cross section *A-A*,

$$M_b = R_B \times 300 = R_A \times 300$$

$$= 1500 \times 300 = 45 \times 10^4 \text{ N-mm}$$

$$\sigma_b = \frac{M_b}{Z_b}$$

$$= \frac{\frac{45 \times 10^4}{\pi d^3}}{32} = \frac{32 \times 45 \times 10^4}{\pi d^3}$$

$$= \frac{1}{d^3}\left[4583662.4\right]$$

For the given stepped shaft, at the change of cross section,

$$\frac{D}{d} = 1.5$$

$$\frac{r}{d} = \frac{0.1d}{d} = 0.1$$

Correspondingly to $\frac{D}{d} = 1.5 \text{ and } \frac{r}{d} = 0.1$

$$K_\sigma = 1.675$$

Taking stress concentration into account,

$$\left.\sigma_b\right)_{\max} A - A = \frac{1}{d^3}\left[4583662.4\right] \times 1.675$$

$$= \frac{1}{d^3}\left[7677634.5\right]$$

At the centre of beam where point load of 3kN acts,

$$M_b = R_B\left(300 + 200\right)$$

$$= 500 R_B$$

$$= 500 \times 1500 = 75 \times 10^4 \text{ N-mm}$$

i.e., $\left.\sigma_b\right)_{B-B} = \dfrac{M_b}{Z_b}$

$$= \frac{\dfrac{75\times10^4}{\dfrac{\pi D^3}{32}}}{} = \frac{75\times10^4\times32}{\pi\left(1.5d\right)^3}$$

$$= \frac{1}{d^3}\left[2263537\right]$$

Therefore, Maximum stress occurs at *A*-*A*, i.e., at change of cross section

Therefore, $100 = \dfrac{1}{d^3}\left[7677634.5\right]$

Therefore, $d = 28.28mm$

$D = 1.5d = 42.42mm$

$r = 0.1d = 2.828mm$

Example 1.41: A flat plate is subjected to a tensile force of 5kN as shown in figure-1.64. the material of plate is grey cast iron having $\sigma_u = 200MPa$. ***Determine the thickness of plate using factor of safety = 2.5.***

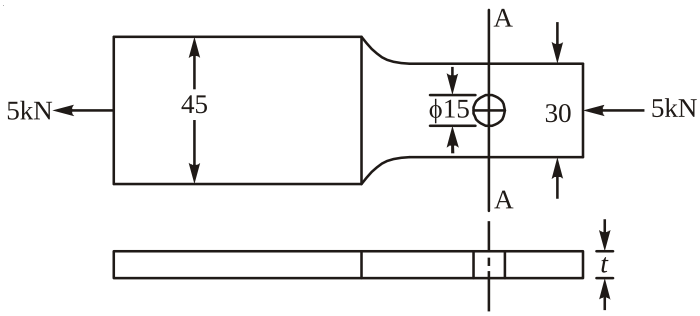

Figure-1.64

Solution:

The plate is subjected to tensile load of 5kN.

The minimum cross sectional area of plate sustaining (at hole) is

$$A = \left(30-d\right)t$$

$$= \left(30-15\right)t = 15t\ mm^2$$

Load, $P = 5kN = 5\times10^3 N$

Therefore, $\sigma_{tensile}\big)_{at\ hole} = \dfrac{P}{A} = \dfrac{5\times10^3}{(30-10)t}$

But, $\sigma_{tensile} = \dfrac{\sigma_{ultimate}}{Factor\ of\ safety}$

$$= \frac{200}{2.5} = 80\ N/mm^2$$

Therefore, $80 = \dfrac{50\times10^3}{(30-10)t}$

Therefore, *t = Thickness of plate = 4.167mm.*

Example 1.42: A rectangular plate 15mm thick made of a brittle material is shown in figure-1.65. Calculate the stress of each of three holes.

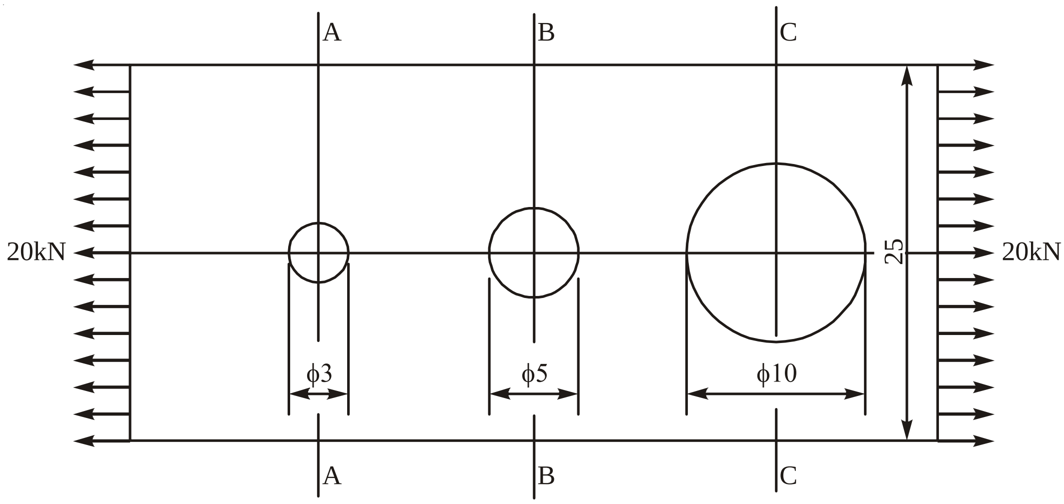

Figure-1.65

Solution:

The plate with holes is under tension

a) Tensile stress at section A-A

$$= \frac{P}{Area\ of\ cross\ section\ at\ A\text{-}A}$$

$$= \frac{20 \times 10^3}{(25-3)15} = 60.606\, N/mm^2$$

b) Tensile stress at section B-B

$$= \frac{P}{Area\ of\ cross\ section\ at\ B\text{-}B}$$

$$= \frac{20 \times 10^3}{(25-5)15} = 66.67\, N/mm^2$$

c) Tensile stress at section C-C

$$= \frac{P}{Area\ of\ cross\ section\ at\ C\text{-}C}$$

$$= \frac{20 \times 10^3}{(25-10)15} = 88.89\, N/mm^2$$

Note: The maximum stress occurs at section C-C where the diameter of hole is maximum.

Example 1.44: The stresses at a point in a member are $\sigma_x = 90\, N/mm^2$, $\sigma_y = 20\, N/mm^2$, $\tau_{xy} = 80\, N/mm^2$, ***and*** $\sigma_{yt} = 300\, N/mm^2$. ***Find the factor of safety by***

a) Maximum shear stress theory.

b) Maximum distortion energy theory.

Solution:

Principal stress

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \qquad \text{... E(2.34)}$$

$$= \left(\frac{90+20}{2}\right) + \sqrt{\left(\frac{90-20}{2}\right)^2 + (80)^2}$$

$$= 55 + 87.32$$

$$= 142.32 MPa\ (tension)$$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \qquad \text{... E(2.35)}$$

$$= 55 - 87.32$$

$$= -32.32 MPa\,(compression)$$

According to Maximum shear stress theory

Maximum shear stress $\tau_{\max}$ induced in the member is

$$\tau_{\max} = Largest\ of\ \frac{1}{2}\left|(\sigma_1 - \sigma_2)\right|,\ \frac{1}{2}\left|\sigma_1\right|,\ \frac{1}{2}\left|\sigma_2\right|$$

$$= \frac{1}{2}\left|142.32 + 32.32\right|,\ \frac{1}{2}\left|142.32\right|,\ \frac{1}{2}\left|32.32\right|$$

$$= \frac{1}{2}\left|174.64\right|$$

Therefore, $\tau_{\max} = 87.32\, N/mm^2$

Therefore, $Factor\ of\ safety = \dfrac{\sigma_{yt}}{2\tau_{\max}}$

$$= \frac{300}{2 \times 87.32} = 1.718$$

According to distortion energy theory,

$$\frac{\sigma_{yt}}{Factor\ of\ safety} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

$$\frac{300}{Factor\ of\ safety} = \sqrt{(142.32)^2 + (-32.32)^2 - \left[(142.32)(-32.32)\right]}$$

Therefore, $Factor\ of\ safety = 1.864$

Example 1.45: A M.S. shaft of 50mm diameter is loaded statically in pure torsion such that the shear stress in the member is 60MPa. If the yield point stress is $400\, N/mm^2$,

determine the factor of safety by

a) ***Maximum shear stress theory.***

b) ***Distortion energy theory.***

Solution:

According to Maximum shear stress theory

For pure torsion, $\tau_{\max} = \sigma_1 = \sigma_2 = 60MPa$

Therefore, $$Factor\ of\ safety = \frac{\sigma_{yt}}{2\tau_{\max}}$$

$$= \frac{400}{2 \times 60} = 3.3$$

According to distortion energy theory,

$$\frac{\sigma_{yt}}{Factor\ of\ safety} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}$$

$$\frac{400}{Factor\ of\ safety} = \sqrt{(60)^2 + (-60)^2 - [(60)(-60)]}$$

$$\frac{400}{Factor\ of\ safety} = 103.923$$

Therefore, $$Factor\ of\ safety = 3.849$$

Example 1.46: Using the following values of principal stress and yield point stress namely $\sigma_1 = 90\,N/mm^2$, $\tau_s = 45\,N/mm^2$, $\tau_{yt} = 300\,N/mm^2$, ***determine the factor of safety using maximum principal stress theory.***

Solution:

According to Maximum shear stress theory

$$\sigma_{\max} = \sigma_1 = 90\,N/mm^2$$

Therefore, $$Factor\ of\ safety = \frac{\sigma_{yt}}{\sigma_{\max}}$$

$$= \frac{300}{90} = 3.33$$

Example 1.47: The stepped plate shown in figure-1.66 is subjected to an axial load fluctuating between 50kN and 100kN. Using yield stress $\sigma_y = 300MPa$ endurance stress $\sigma_{-1} = 150MPa$, taking stress concentration into account, determine the dimension h based on a factor of safety = 3. The maximum width of plate is 120mm.

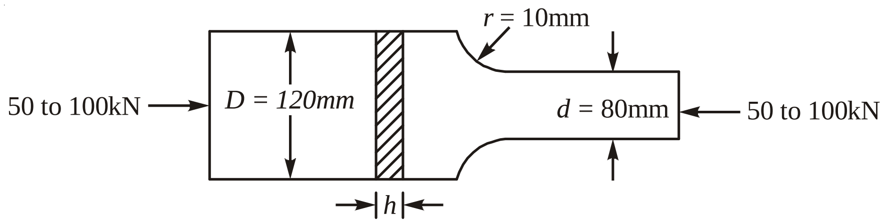

Figure-1.66

Solution:

For fluctuating axial load, Soderberg's equation is

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_y} = 1 \qquad \text{... E(5.40)}$$

To find σ_a, the stress amplitude

$$\sigma_a = \frac{F_a}{A}$$

Where, $$F_a = \frac{F_{\max} - F_{\min}}{A}$$

$$= \frac{100 \times 10^3 - 50 \times 10^3}{2} = 25 \times 10^3$$

A = Area of cross section member

$$= dh = 80 \times h = 80h$$

Therefore, $$\sigma_a = \frac{25 \times 10^3}{80h} = \frac{312.5}{h}$$

To find σ_m, the mean stress

$$\sigma_m = \frac{F_m}{A}$$

Where, $$F_m = \frac{F_{\max} + F_{\min}}{2}$$

$$= \frac{100 \times 10^3 + 50 \times 10^3}{2} = 75 \times 10^3$$

Therefore, $\sigma_m = \frac{75 \times 10^3}{80h} = \frac{937.5}{h}$

To find σ_{-1d}, the endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

Where, $\sigma_{-1} = 150 MPa$

$n_{-1d} = K_{sz} K_{sr} K_l \, n_{-1a}$

$= 0.85 \left(\frac{1}{e_{sr}} \right) 0.7 \times 3$

Assuming $\sigma_u = 2\sigma_{-1}$

$= 2 \times 150 = 300 MPa$

$e_{sr} = 0.86$ (From figure-5.3 MDH)

Therefore, $n_{-1d} = 2.076$

$\sigma_{-1d} = \frac{150}{2.076} = 72.25 \, N/mm^2$

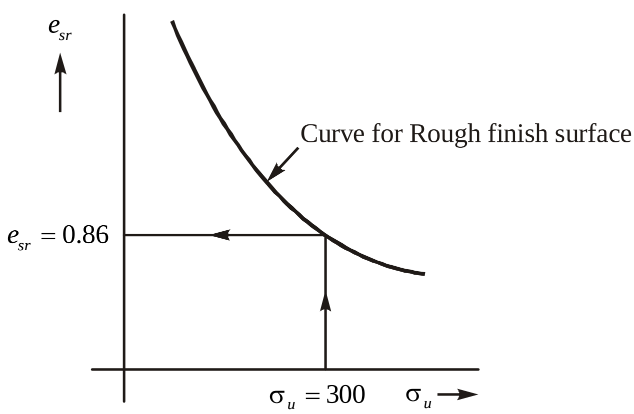

Figure-5.3 MDH

To find σ_{yd}, the design yield stress

$$\sigma_{yd} = \frac{\sigma_y}{n_{yd}}$$

Where, $\sigma_y = 300 MPa$

$$n_{yd} = K_{sz} K_{\sigma a} n_{ya}$$

$$= 0.85 \times K_{\sigma a} \times 3$$

To find $K_{\sigma a}$, the stress concentration factor

For the given plate, $\frac{D}{d} = \frac{120}{80} = 1.5$

$$\frac{r}{d} = \frac{10}{80} = 0.125$$

Corresponding to $\frac{D}{d} = 1.5 \; and \; \frac{r}{d} = 0.125$ From Figure (4.24A/4.30 MDH)

Stress concentration factor, $K_\sigma = 1.95$

Therefore, $n_{yd} = 0.85 \times 1.95 \times 3 = 4.9725$

Therefore, $\sigma_{yd} = \frac{300}{4.9725} = 60.33 \, N/mm^2$

Substituting the above values in Soderberg's equation,

$$\frac{\frac{312.5}{h}}{72.25} + \frac{\frac{937.5}{h}}{60.33} = 1$$

Therefore, $h = Thickness \; of \; plate = 19.86mm. \; Adopt \; h = 20mm$

Example 1.48: A stepped plate shown in figure-1.67 is subjected to a load varying from 100kN to 200kN. Taking $\sigma_y = 300 \, N/mm^2$, $\sigma_{-1} = 250 \, N/mm^2$, $\sigma_u = 500 \, N/mm^2$, determine the dimensions of plate based on factor of safety = 3.

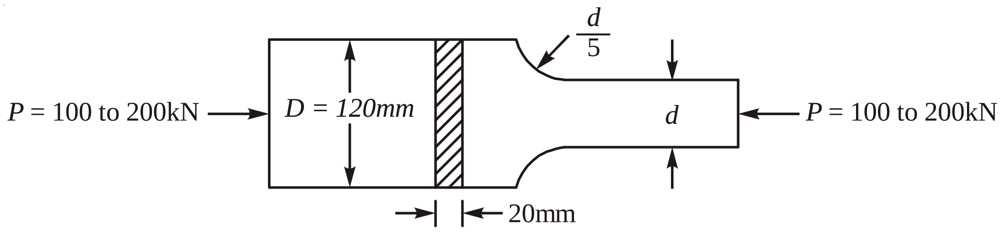

Figure-1.67

Solution:

The plate is subjected to fluctuating axial load.

Therefore, Soderberg's equation for axial load is

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad \text{... E(5.40)}$$

Where, $$\sigma_a = \frac{F_a}{A}$$

$$F_a = \frac{F_{\max} - F_{\min}}{2}$$

$$= \frac{200\times10^3 - 100\times10^3}{2} = 50\times10^3$$

$$F_{\max} = 200kN = 200\times10^3 N$$

$$F_{\min} = 100kN = 100\times10^3 N$$

Area of cross section,

$$A = dh = 20d$$

Therefore, $$\sigma_a = \frac{50\times10^3}{20d} = \frac{2.5\times10^3}{d}$$

To find σ_m***, the mean stress***

$$\sigma_m = \frac{F_m}{A}$$

Where, $$F_m = \frac{F_{\max} + F_{\min}}{2}$$

$$= \frac{200\times10^3 + 100\times10^3}{2} = 150\times10^3 N$$

Therefore, $\sigma_m = \frac{150\times10^3}{20d} = \frac{7.5\times10^3}{d}$

To find σ_{-1d}, the endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

Where, $\sigma_{-1} = 250\, N/mm^2$

$$n_{-1d} = K_{sz}K_{sr}K_l\, n_{-1a}$$

$$= 0.85\left(\frac{1}{e_{sr}}\right)0.7\times3$$

$e_{sr} = 0.83$ (From figure-5.3 MDH)

$$= 0.85\left(\frac{1}{0.83}\right)0.7\times3 = 2.151$$

Therefore, $\sigma_{-1d} = \frac{250}{2.151} = 116.25\, N/mm^2$

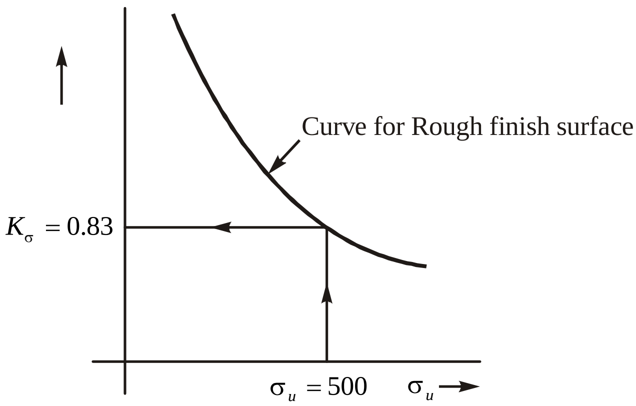

Figure-5.3 MDH

To find σ_{yd}, the design yield stress

$$\sigma_{yd} = \frac{\sigma_y}{n_{yd}}$$

Where, $\sigma_y = 300\,N/mm^2$

$$n_{yd} = K_{sz}K_{\sigma a}n_{ya}$$

$$= 0.85\times K_{\sigma a}\times 3 = 2.55\times K_{\sigma a}$$

To find $K_{\sigma a}$, the stress concentration factor

For the given plate, $\dfrac{D}{d} = \dfrac{1.5d}{d} = 1.5$

$$\frac{r}{d} = \frac{\frac{d}{5}}{d} = 0.2$$

Corresponding to $\dfrac{D}{d} = 1.5 \text{ and } \dfrac{r}{d} = 0.2$ From Figure (4.24A/4.30 MDH)

Stress concentration factor, $K_{\sigma} = K_{\sigma a} = 1.725$

Therefore, $n_{yd} = 2.55\times 1.725 = 4.4$

Therefore, $\sigma_{yd} = \dfrac{300}{4.4} = 68.18\,N/mm^2$

Substituting the above values in Soderberg's equation,

$$\frac{\frac{2.5\times 10^3}{d}}{116.25} + \frac{\frac{7.5\times 10^3}{d}}{68.28} = 1$$

$$d = 131.35mm$$

Therefore, Dimensions of plate are,

$$d = 131.35mm$$

$$D = 2d = 262.7mm$$

$$r = \frac{d}{5} = \frac{131.25}{5} = 26.25mm$$