Chapter 2

DESIGN FOR STATIC AND IMPACT STRENGTH

Strength is a property or characteristics of a material or of a mechanical element. The strength of an element depends on the choice, the treatment and the processing of the material. It is to be noted that strength may be inherent in a material or may result from the treatment in processing employed. Since strength is a property of material, it is completely independent of whether the part is subjected to or not subjected to a load or a force.

Static strength

In designing any machine element. The designer should have the strength results of the material chosen. The tests should have been made on specimens having the same size, same surface finish and the same heat treatment as the element the designer proposes to design. It is to be noted that tests should be made with exactly the same loading conditions as the part will experience in service. That is if the part designed is to experience a bending load, it is to be tested in bending. Similarly if it is subjected to combined bending and torsional loads, it is to be tested under combined bending and torsional loads and so on.

Tests conducted for strength should provide useful and processed information to the designer so that the part designed by him will withstand the loads it is expected to.

Static loads and factor of safety

A static load is a stationary force or moment acting on a member. To be stationary, the force or moment must have an unchanging magnitude, unchanging point or points of application and an unchanging direction. A static load may be an axial tensile load, compressive load, bending load or torsional load or any combination of these. The term factor of safety is applied to factor used to evaluate the safeness of a member.

Consider a mechanical element subjected to some effect designated by F. This F is a general term and it can be a force, a moment and so on. If F is increased, finally it becomes so large that any additional small increase would permanently impair the ability of the member to perform its function. If this limiting value or ultimate value of F is designated by F_u then factor of safety is given by

$$\text{Factor of safety } n = \frac{F_u}{F}$$

When $F_u = F$, factor of safety becomes unity and there is no safety at all for the member.

Factor of safety or design factor is used the engineers and designers to account separately for the uncertainties that may occur in the strength of a part with the loads acting on the member.

If n_1 and n_5 are the factors of safety used for loads and strength respectively. Then the overall factor of safety or total factor of safety '*n*' is given by

$n = n_l n_s$

For brittle materials such as Cast Iron, which do not have an yield point, factor of safety is based on ultimate stress and hence.

$$Factor\ of\ safety\ n = \frac{\sigma_u}{\sigma_d}$$

Where $\sigma_u = Ultimate\ stress$

$\sigma_d = Design$ stress

For ductile materials such as steel, factor safety is based on yield stress.

Therefore, $$Factor\ of\ safety\ n = \frac{\sigma_y}{\sigma_d}$$

Where, $\sigma_y = yield\ stress$

Theories of failure

In designing parts to resist failure, we assume that the internal stresses do not exceed the strength of material. The design of ductile materials (such as steel) is based on yield strength and that of brittle materials (such as Cast iron) are based ultimate stress since they do not have an yield point.

Maximum normal stress theory (Also called Rankine theory)

Statement: Maximum normal stress theory states that "failure occurs whenever the largest principal stress equals the strength"

If σ_1, σ_2 and σ_3 are the three principal stresses for any stress state with $\sigma_1 > \sigma_2 > \sigma_3$, then if yielding is the criteria of failure, according to this theory, failure occurs when

$$\sigma_1 = \sigma_{yt} \text{ or } \sigma_3 = \sigma_{-yc}$$

Where σ_{yt} and σ_{yc} are the yield strength in tension and compression respectively.

$$\sigma_{\max} \leq \text{allowable stress for ductile materials}$$

$$\sigma_{\max}\,(tensile) \leq \frac{\sigma_{yt}}{f.o.s};$$

$$\sigma_{\max}\,(compressive) \leq \frac{\sigma_{yt}}{f.o.s}$$

For brittle materials,

$$\sigma_{\max}(tensile) \leq \frac{\sigma_{ut}}{f.o.s};$$

$$\sigma_{\max}(compressive) \leq \frac{\sigma_{uc}}{f.o.s}$$

Note: ***For ductile materials, the values of yield stress in tension and compression are usually same and hence*** $\sigma_{ut} = \sigma_{uc}$

For brittle materials (such as cast iron), failure occurs when

$$\sigma_1 = \sigma_{ut} \text{ or } \sigma_3 = \sigma_{-uc}$$

When σ_{ut} and σ_{uc} are the ultimate tensile and compressive stress respectively.

For pure torsion, $\sigma_1 = \tau = \sigma_3$ and $\sigma_2 = 0$. This theory predicts that the part would fail

$$\sigma_3 = 0 \text{ When } \tau = \sigma_y$$

This theory is better understood with the following examples.

Example 2.1: A material has an yield strength of 600MPa. Determine the factor of safety using maximum normal stress theory. Use $\sigma_1 = 420MPa$ ***,*** $\sigma_2 = 420MPa$ ***and*** $\sigma_3 = 0$ $\sigma_3 = 0$.

Solution:

According to maximum normal stress theory,

$$\sigma_{\max} \leq \sigma_1 = \frac{\sigma_{yt}}{f.o.s}$$

Therefore, $$Factor\ of\ safety\ n = \frac{\sigma_{yt}}{maximum\ Principal\ stress}$$

$$= \frac{600}{\left[420 - (-180)\right]} = 1$$

Here, $\sigma_1 = \sigma_2 = 420 MPa$

Therefore, $$n = \frac{\sigma_{yt}}{\sigma_1} = \frac{600}{420} = 1.4286$$

Example 2.2: A ductile material has a yield stress of 600MPa. Using the following stress state, $\sigma_1 = 420 MPa$ ***,*** $\sigma_2 = 180 MPa$ ***and*** $\sigma_3 = 420 MPa$ ***, determine the factor of safety.***

Solution:

Here, $$|\sigma_1| = |\sigma_3| > |\sigma_2| \quad \sigma_{max} \leq \sigma_1 = \frac{\sigma_{yt}}{f.o.s}$$

Therefore, $$Factor\ of\ safety\ n = \frac{\sigma_{yt}}{\sigma_1\ or\ \sigma_3}$$

$$= \frac{\sigma_{yt}}{\sigma_1} = \frac{600}{420} = 1.4286$$

Maximum shear stress theory

Statement:

The maximum shear theory states that "yielding begins whenever the maximum shear stress in the member equals the maximum shear stress in a tension test specimen of the same material when the specimen begins to yield".

Assuming $\sigma_1 > \sigma_2 > \sigma_3$, the maximum shear stress theory predicts that failure will occur whenever

$$\tau_{max} = \frac{\sigma_y}{2};\ \sigma_1 - \sigma_3 = \sigma_y$$

Because $\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$

According to this theory, $\tau_{\max}$ = *yeild shear stress = 0.5*σ_y.

Example 2.3: A material has an yield strength of 600MPa. The following stress state exists at a point in the number

Solution:

$\sigma_1 = 420MPa$, $\sigma_2 = 0MPa$ and $\sigma_3 = -180MPa$. Determine the factor of safety.

1) Factor of safety

$$n = \frac{\sigma_{yt}}{\sigma_1 - \sigma_3}$$

$$= \frac{600}{\left[420-(-180)\right]} = 1 \quad \sigma'$$

2) For the same material, if $\sigma_1 = 0$, $\sigma_2 = 180MPa$ ***and*** $\sigma_3 = -420MPa$

$$n = \frac{\sigma_{yt}}{\sigma_1 - \sigma_3}$$

$$= \frac{600}{0-(-420)} = 1.4286$$

Distortion energy theory (Also called shear energy theory or Von Mises-Hencky theory)

Statement:

This theory state that "failure occurs when the strain energy of distortion per unit volume at any point becomes equal to the strain energy of distortion per unit volume in a standard tension test specimen, when yielding starts"

This theory is best suited for ductile materials and gives accurate results. In the tension test,

$$\sigma_1 = \sigma_x = \sigma_{yt}\,;\ \sigma_2 = \sigma_3 = 0$$

For a triaxial stress state.

We hav e $\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$

Where $\sigma' = \frac{\sigma_{yt}}{f.o.s}$

$$\left(\frac{\sigma_{yt}}{factor\ of\ safety}\right)^2 = \sigma^2{}_1 + \sigma^2{}_2 + \sigma^2{}_3 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1$$

For a two dimensional case, $\sigma_2 = 0$

Therefore, $\left(\frac{\sigma_{yt}}{factor\ of\ safety}\right)^2 = \sigma^2{}_1 + \sigma^2{}_2 - \sigma_1\sigma_2$

$$= \sigma^2{}_x + \sigma^2{}_y + 3\tau^2{}_{xy}$$

Example 2.4: At a point in a material having $\sigma_{yt} = 600MPa$ ***,the principal stresses are,*** $\sigma_1 = 0$ **,** $\sigma_2 = -180MPa$ ***and*** $\sigma_3 = -180MPa$***. Determine the factor of safety.***

Solution:

Von-Mises stress σ' is given by

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$= \sqrt{\frac{(0+18)^2 + (-180+420)^2 + (-420+0)^2}{2}}$$

Now $\sigma' = \frac{\sigma_{yt}}{f.o.s}$

Therefore, $Factor\ of\ safety\ n = \frac{\sigma_{yt}}{\sigma'}$

$$= \frac{600}{364.97} = 1.64$$

Example 2.5: Determine the factor of safety for a material having $\sigma_{yt} = 600MPa$ having the following values of principal stresses, $\sigma_1 = 420MPa$, $\sigma_2 = 420MPa$ and $\sigma_3 = 0$

Solution:

Von Mises stress σ' is given by

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

Substituting, $\sigma_1 = 420MPa$, $\sigma_2 = 420MPa$ and $\sigma_3 = 0$ we get,

$$\sigma' = \sqrt{\frac{(420 - 420)^2 (420 - 0)^2 + (0 - 420)^2}{2}}$$

$$\sigma' = 420MPa \quad \sigma_{uc} >> \sigma_{ur}$$

$$Factor\ of\ safety = \frac{\sigma_{yt}}{\sigma'}$$

$$= \frac{600}{420} = 1.426$$

Failure of Ductile material:

Predictions using maximum normal stress theory do not agree with experimental results on and many a time give results the unsafe side. For a biaxial state, one of the principal stress is zero, Let σ_A and σ_B be the remaining non zero stresses.

When the three theories of failure are plotted on a σ_A, σ_B co-ordinate system, give the graph as shown in the figure 2.1

It has been found from experimental results that Distortion Energy theory Predicts yielding with greatest accuracy in all the four quadrants. Based on the assumption that Distortion energy theory as the correct one, because its graph is inside the Distortion energy ellipse.

The maximum stress theory is the same as that of maximum shears stress theory in the first and third quadrants. However the graph of the maximum normal stress theory lies outside the distortion energy theory in the second and fourth quadrants. Hence, it is not worth to use maximum normal stress theory as there are chances of predicating safety when no safety exists.

It can be finally concluded that

i) Maximum shear stress theory may be employed in situations when

a) Dimensions of the component need not be held very close.

b) A quick determination of size is the requirement

c) Factor of safety can be used generously.

ii) Since distortion energy theory predicts failure very accurately, it is used when

a) Factor of safety is to be very close limits

b) During the investigation of part failure.

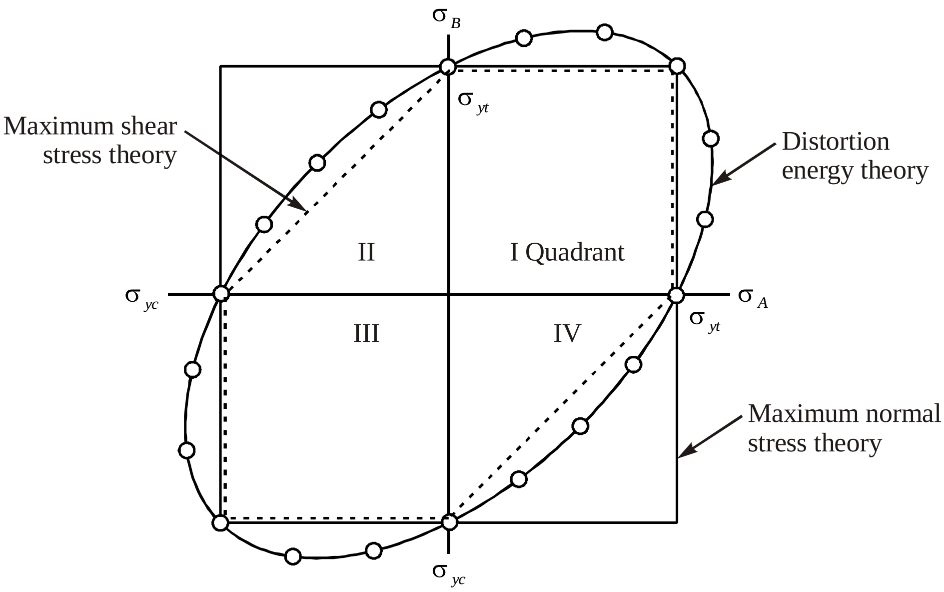


Figure2.1

Failure of Brittle materials

For brittle materials the plot of stress versus strain is a continuous line to the failure point. Failure occurs by facture and the materials do not have a yield point. The compressive strength is many times higher than the tensile strength. i.e., $\sigma_{uc} >> \sigma_{ur}$

The maximum normal stress theory and coulomb-Mohr theory are used for predict failure of brittle materials. It is to be noted that the test points used for ductile material are to be changed to σ_{ut} and σ_{uc} (in place of σ_{yt} and σ_{yc}).

If σ_1, σ_2 and σ_3 are the principal stresses at a point in a material then,

$$\frac{\sigma_1}{\sigma_{ut}} - \frac{\sigma_3}{\sigma_{uc}} = 1 \text{ where } \sigma_1 \geq 0 \text{ and } \sigma_3 \leq 0$$

Stress Concentration

When the cross section of a component changes abruptly as in the case of components having hole, notch or groove or where a small section joints a larger one, the stress in the piece fails to follow the elementary equations such as $\sigma_1 = \frac{F}{A}$; where $F = load$ and $A = area\ of\ cross\ section.$

Consider a flat plate of width *b*, thickness *h* subjected to a pull of *F* Newton. Due to *F*, the tensile stress in the plate is given by $\sigma_1 = \frac{F}{bh}$. The magnitude of σ_1 is uniform at all sections A-A, B-B, C-C, etc. This is indicated by the force flow lines 1-1, 2-2, 3-3, etc. running parallel to load axis (or parallel to edge of plate) Refer figure-2.2.

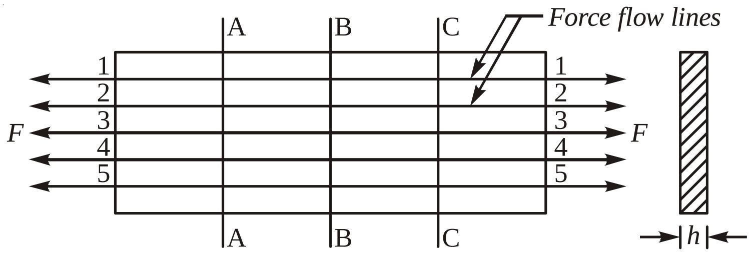


Figure-2.2

If a hole of diameter *d* is drilled at the centre (in the same plate) and subjected to load *F*, then the force flow lines near the discontinuity (hole) gets crowded and no longer the lines run parallel to the edge. Refer figure-2.3.

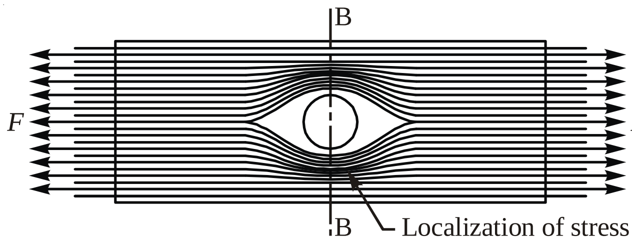


Figure-2.3

Figure-2.4 shows the same plate in which two additional small holes are drilled one on either side of larger hole. Due to *F*, the force flow lines move apart and the crowding of these lines at section B-B get reduced i.e., the stress accumulation is reduced to some extent.

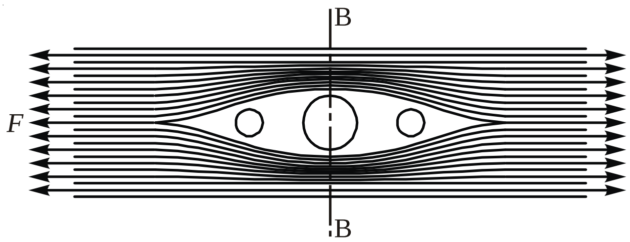


Figure-2.4

Figure-2.5 shows a stepped bar subjected to tensile load *F*. The cross section CD where the diameter changes abruptly from *D* to *d*, the force flow lines gets crowded and result in increased stress.

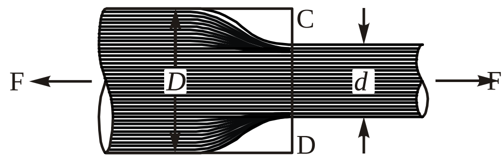


Figure-2.5

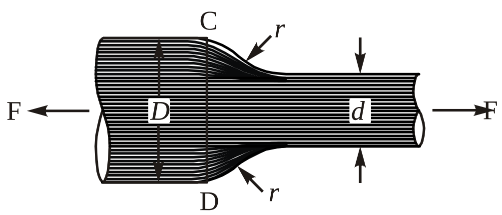


Figure-2.6

Figure-2.6 shows the same shaft provided with a fillet of radius r. Here the crowding of force flow lines is reduced to some extent at section CD.

In such cases the fibres closer to the abrupt change in cross section are affected more than those farther from this place. This phenomenon is called stress concentration or localized stress. The change of section is called a discontinuity or a stress raiser.

Stress concentration may be defined as the accumulation of stress at a point due to the presence of stress raisers such as notches, grooves, etc.

Stress concentration factor K_σ is often referred to as the theoretical stress concentration factor. It is derived analytically based on the assumptions of elasticity and Hooke's law.

Stress concentration factor is defined as the ration of maximum stress to nominal stress.

i) For tension, stress concentration factor is given by,

$$K_\sigma = \frac{\sigma_{\text{max}}}{\sigma_{\text{nominal}}}$$

Where, $$\sigma_{\text{nominal}} = \frac{Load}{Minimum\ area\ of\ cross\ section}$$

ii) For bending stress, concentration factor is given by,

$$K_\sigma = \frac{\sigma_{b\,\max}}{\sigma_{b\,\text{nominal}}}$$

Where, $\sigma_{b\ \text{nominal}} = \dfrac{M_b}{Z_{b\min}} = \dfrac{M_b}{\dfrac{\pi d^3}{32}}$ for a stepped shaft having minimum diameter *d* subjected to bending and $Z_b = \dfrac{1}{c} =$ section modulus $= \dfrac{\pi d^3}{32}$

iii) For torsion, the stress concentration factor is given by,

$$K_\tau = \frac{\tau_{\max}}{\tau_{\text{nominal}}},\ \text{where}\ \tau_{\text{nominal}} = \frac{M_t}{Z_{t\min}}$$

Where, $Z_{t\min} = \dfrac{\pi d^3}{16,}$ where *d* is the least diameter of stepped shaft subjected to torsion.

The effect of stress raiser depends on three factors namely

i) Size and type of discontinuity.

ii) Material properties and

iii) Type of load.

The stress raisers most commonly found in machine elements are holes, grooves, fillets, notches and key ways. A clear concept of stress concentration helps the designer to visualize the probable stress concentration in the parts being designed.

Figure-2.7 shows the stress concentration caused by the presence of notches (*v* and circular)

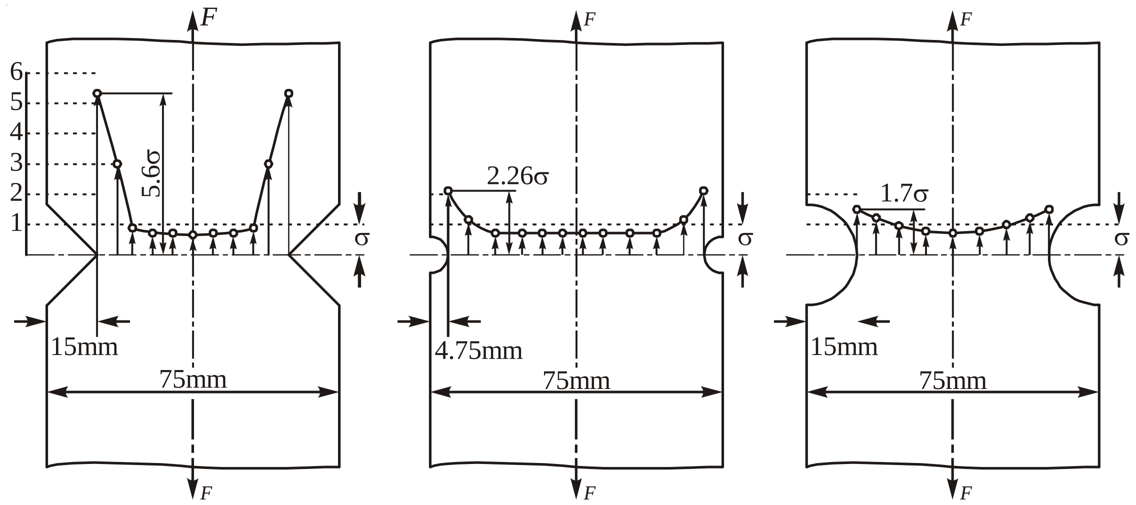


Figure-2.7: Stress concentration caused by notches

Figure-2.8 shows the stress concentration at the root of a spur gear tooth caused due to tangential load *F*.

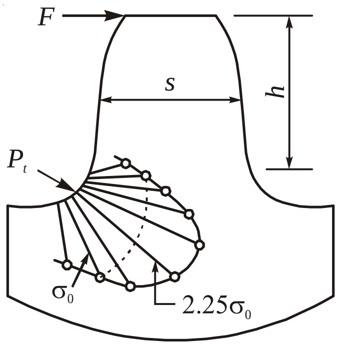


Figure-2.8: Stress concentration at the root of a spur gear tooth

Determination of stress concentration factors

The following are some of the methods used to determine the stress concentration factors.

a) Photo Elasticity

Photo elasticity is a very dependable and widely used method to determine stress at a point in a material. In this method, a transparent material having double refraction properties when stressed is cut into the same shape as the part whose stresses are to be determined. The model is placed between the polarizer and analyzer in case of plane polariscope and between first and second quarter wave plates in a loading frame of a circular polariscope. When a beam of polarized light is passed through the specimen and loaded, fringes of coloured light appear at the point of maximum stress. The number of fringes produced is directly proportional to the load applied.

b) Finite Element Technique (or Finite Element Method)

Finite element technique called as FET in short is a very powerful and new approach made possible with the use of computer and computer aided design.

The finite elements used in the analysis may be lines, triangles or any convenient geometric shapes. The member to be analyzed is first divided into a large member of finite elements of different sizes. Commencing from a known loading and boundary, any configuration and constraints, a computer analysis is made and iterations are made till all conditions are satisfied. Knowledge of matrix mathematics and linear elasticity are the basic requirements for this analysis.

c) Grid Method

The grid method consists of a drawing a grid of lines on the part or on the model of it. Two sets of equally spaced lines at right angles to each other on a number of concentric circles intersected by radial lines are used to form the grid. The part is then loaded. The change in spacing of lines is used to map the strain. Since the strains are small, they are magnified. After the strains are obtained, the stresses are calculated using bi-axial stress relations. It is to be noted that food results are possible only when the use of lines accurately scribed and the measurement of strains are accurate.

d) Brittle Coating Technique

In brittle coating technique, a thin layer of lacquer of stress coat is applied on the part under carefully controlled temperature and humidity conditions. After the lacquer has dried, sufficient load is applied on the part so as to cause tiny cracks at areas of high tensile stresses. Since the lacquer is brittle and fails in tension, cracks are formed at right angles to the direction of load. The first crack is formed on the part at the region of highest tensile stress. When loading is increased, other cracks are formed indicating the areas of lower stresses. The major advantage of this method is that it can be applied to components of irregular shape.

e) Brittle Model Method

As the name indicates, this method employs a very brittle material such as plaster for the calib5ration specimen and model of the part to be investigated for stress concentration. Each specimen is loaded till fracture occurs and the respective loads are compared to obtain stress concentration factor.

f) Stain Gauge Method

In this method, electrical or mechanical strain gauges are used to measure strains. The stresses are then computed using bi-axial stress equation or tri-axial stress equations depending on whether strain gauge or strain gauge rosette employed. It is to be noted that proper preparation of surface of the specimen where the strain gauges are to be mounted and proper installation of strain gauges at the point of interest give correct results of strain.

Example 2.6: A steel plate of width 50mm, having a hole of diameter 10mm drilled at its centre is subjected to a tensile load of 1kN. Determine the maximum stress induced in the member taking stress concentration into account.

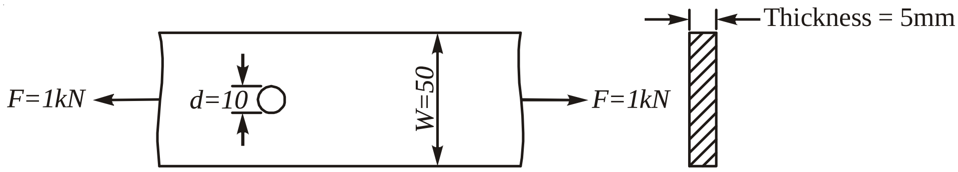


Figure-2.9

Solution:

For a flat plate with a hole at its centre subjected to tensile load, referring figure-4.5 MDH.

Stress concentration factor

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

Therefore, $\sigma_{\max} = K_\sigma \sigma_{\text{nominal}}$

Where, $\sigma_{\max}$ = Maximum stress; N/mm^2

K_σ = Stress concentration factor

σ_{nominal} = Nominal stress $= \dfrac{F}{(W-d)h}$

F = load $= 1kN = 1000N$

W = width of plate $= 50mm$

d = diameter of hole drilled $= 10mm$

h = plate thickness $= 5mm$

Therefore, $$\sigma_{\text{nominal}} = \frac{1000}{(50-10)5} = 5\,N/mm^2$$

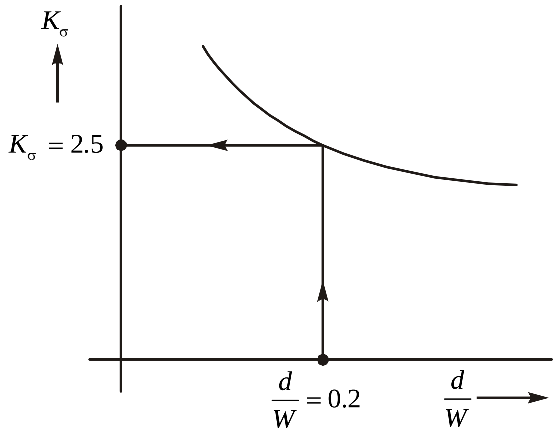


Figure-2.10

To find K_σ, the stress concentration factor

For the given plate,

$$\frac{d}{W} = \frac{10}{50} = 0.2$$

Corresponding to $\frac{d}{W} = 0.2$ stress concentration factor $K_\sigma = 2.5$ [Figure 4.8/4.5 MDH]

$$\sigma_{\max} = K_\sigma \sigma_{\text{nominal}}$$

$$= 2.5 \times 5 = 12.5\, N/mm^2$$

Therefore, Maximum stress induced in the member $= 12.5\, N/mm^2$

Example 2.7: A steel plate of width 50mm and thickness 10mm with a hole of 10mm diameter drilled at its centre is subjected to a tensile load F. Determine the load F, the member can withstand considering stress concentration and limiting the maximum stress in the member to $25\, N/mm^2$.

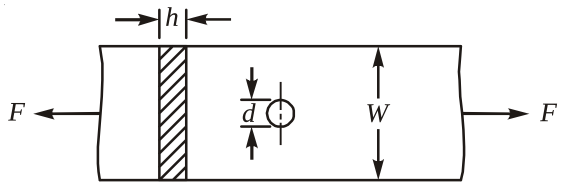


Figure-2.11

Solution:

Stress concentration factor

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

For the given plate,

$$d = diameter\ of\ \ hole = 10mm$$

$$W = width\ of\ \ plate = 50mm$$

$$\frac{d}{W} = \frac{10}{50} = 0.2$$

Corresponding to $\frac{d}{W} = 0.2$ stress concentration factor $K_\sigma = 2.5$ [Figure 4.8/4.5 MDH]

$$\sigma_{\max} = 25\, N/mm^2\,(given)$$

$$\sigma_{\text{nominal}} = \frac{F}{(W-d)h}$$

$$= \frac{F}{(50-10)10} = \frac{F}{400}$$

From, $$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

$$2.5 = \frac{25}{\frac{F}{400}}$$

Therefore, $$F = \frac{10000}{2.5} = 4000N = 4kN$$

Therefore the maximum load *F* the member can withstand = 4kN.

Example 2.8: A flat steel plate of width 50mm, thickness = 10mm having a hole of diameter 5mm drilled at its centre is subjected to a bending moment of 800 N-mm. Determine the maximum bending stress induced in the member taking stress concentration into account.

Solution:

For bending, stress concentration factor

$$K_\sigma = \frac{\sigma_{b\,\max}}{\sigma_{b\,\text{nominal}}}$$

Therefore, $\sigma_{b\max} = K_\sigma \sigma_{b\text{nominal}}$

$\sigma_{b\max} =$ Maximum bending stress

$K_\sigma =$ Stress concentration factor

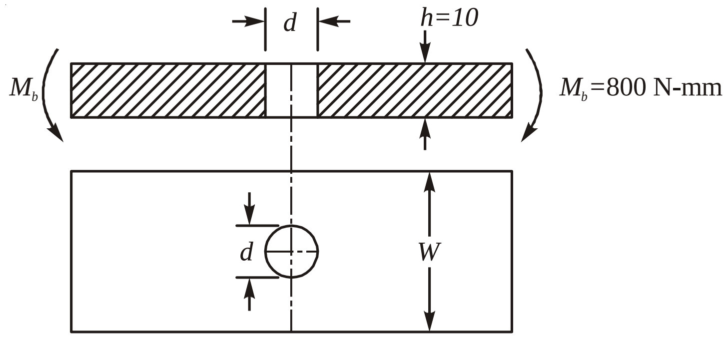


Figure-2.12

For the given plate,

Width of plate,	W = 50mm.
Diameter of hole,	d = 5mm.
Thickness of plate,	h = 10mm.

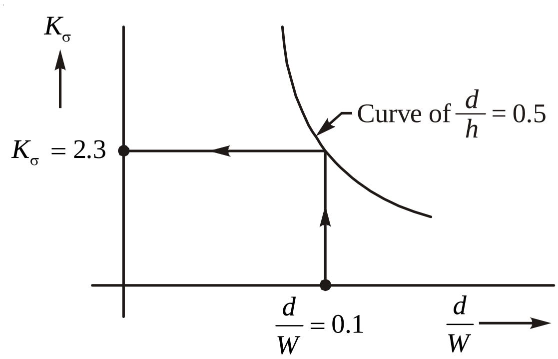


Figure-2.13

Therefore, $\frac{d}{W} = \frac{5}{50} = 0.1$

$$\frac{d}{h} = \frac{5}{10} = 0.5$$

Corresponding to $\frac{d}{W} = 0.1$ and $\frac{d}{h} = 0.5,$ stress concentration factor $K_\sigma = 2.3$

$$\sigma_{b\,\text{nominal}} = \frac{6M_b}{(W-d)h^2} \qquad \text{(Figure 4.6 MDH)}$$

$$= \frac{6 \times 800}{(50-5)10^2} = 1.066\, N/mm^2$$

Therefore, $\sigma_{b\max} = 2.3 \times 1.066 = 2.4518\, N/mm^2$

Example 2.9: A stepped shaft stepped down from 50mm to 25mm with a fillet radius of 5mm is subjected to a torsional moment (torque) of 100 N-m. Determine the maximum shear stress induced in the shaft taking stress concentration into account.

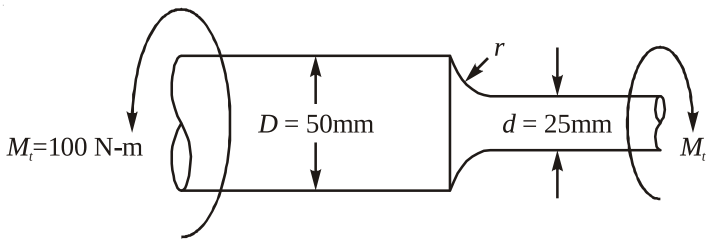


Figure-2.14

Solution:

Stress concentration factor K_τ for member subjected to torsion is given by,

$$K_\tau = \frac{\tau_{\max}}{\tau_{\text{nominal}}}$$

Therefore, $\sigma_{\max} = K_\tau \times \tau_{\text{nominal}}$

Where, $\tau_{\max}$ = maximum shear stress

To find K_τ, the stress concentration factor

For the given shaft,

$$\frac{D}{d} = \frac{50}{25} = 2$$

$$\frac{r}{d} = \frac{5}{25} = 0.2$$

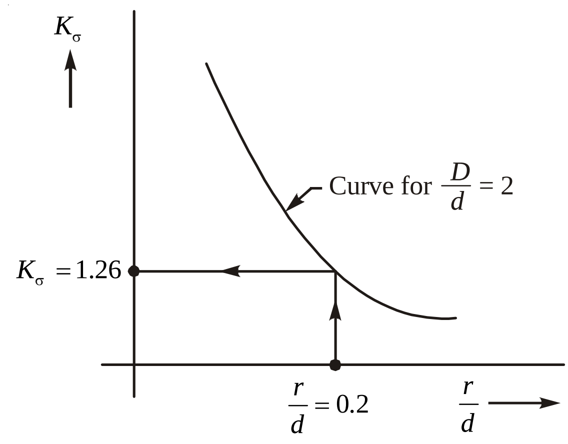


Figure-2.15

Corresponding to $\dfrac{D}{d} = 2$ and $\dfrac{r}{d} = 0.2,$ the stress concentration factor $K_\tau = 1.26$

[Figure 4.19A/4.23MDH]

Therefore, $\tau_{\text{nominal}} = \dfrac{M_t}{Z_t}$

$$= \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16M_t}{\pi d^3}$$

$$= \frac{16\left(100\times10^3\right)}{\pi\left(25\right)^3} = 32.59\,N/mm^2$$

$$\tau_{\max} = 1.26\times32.59 = 41.06\,N/mm^2$$

Therefore, *The maximum shear stress induced in the member* $= 41.06\,N/mm^2$.

Example 2.10: If the stepped shaft in the previous example is subjected to a bending moment of 100 N-m, determine the maximum bending stress in the member taking stress concentration into account.

Solution:

Refer [Figure 4.21A/4.26 MDH]
For bending,
Stress concentration factor

$$K_{\sigma} = \frac{\sigma_{b\max}}{\sigma_{b\text{ nominal}}}$$

Therefore, σ_b = Maximum bending stress = $K_{\sigma}\sigma_{b\text{nominal}}$

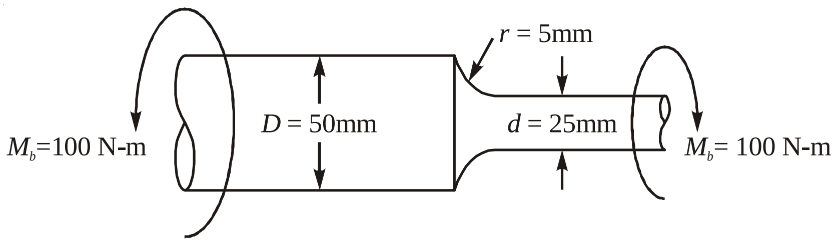


Figure-2.16

To find K_{σ}, the stress concentration factor

For the given stepped shaft,

$$\frac{D}{d} = \frac{50}{25} = 2$$

$$\frac{r}{d} = \frac{5}{25} = 0.2$$

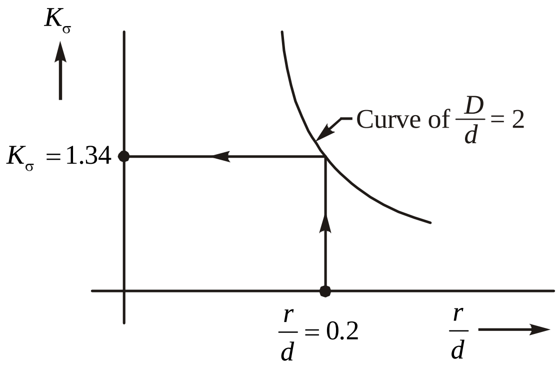


Figure-2.17 [Figure 4.21 MDH]

Corresponding to $\frac{D}{d} = 2$ and $\frac{r}{d} = 0.2,$ the stress concentration factor $K_{\tau} = 1.44$ (approximate)

To find $\sigma_{b\text{nominal}}$

$$\sigma_{b\text{nominal}} = \frac{M_b}{Z_b}$$

Where, $$Z_b = \text{section modulus} = \frac{1}{c} = \frac{\pi d^3}{32}$$

$$\sigma_{b\text{nominal}} = \frac{M_b}{\frac{\pi d^3}{32}} = \frac{32M_b}{\pi d^3}$$

$$= \frac{32 \times 100 \times 10^3}{\pi (25)^3} = 65.19\ N/mm^2$$

$$\sigma_{b\max} = 1.44 \times 65.19 = 93.87\ N/mm^2$$

Therefore, *The maximum bending stress induced in the member* $= 93.87\ N/mm^2$.

Example 2.11: An infinite plate with an elliptical cutout having major diameter 50mm and minor diameter 25mm is subjected to a tensile load F. Determine stress concentration factor when

i) The load is at right angle to the major axis.

ii) The load is parallel to the major axis. [VTU, January 2008/2009]

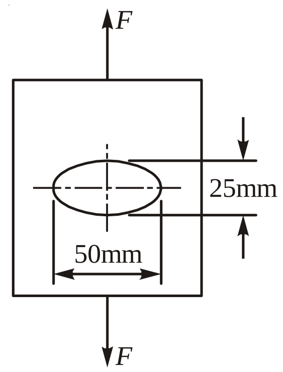


Figure-2.18

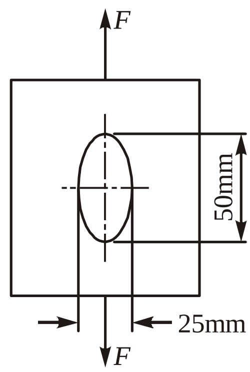


Figure-2.19

Solution:

Case (i)

Referring figure-4.4b (MDH), stress concentration factor K_σ for a plate with the load axis perpendicular to major axis of ellipse is given by,

$$K_\sigma = 1 + 2\frac{a}{b} \quad \text{... E(4.9a)}$$

Where, $2a = \text{Major axis} = 50mm,\ a = 25mm$

$2b = \text{Minor axis} = 25mm,\ b = 12.5mm$

Therefore, $$K_\sigma = 1 + 2\frac{25}{12.5} = 5$$

Case (ii) Stress concentration factor K_σ for a plate with load axis parallel to the major axis of ellipse is given by,

$$K_\sigma = 1 + 2\frac{b}{a} \quad \text{... E(4.9c)}$$

Therefore, $$K_\sigma = 1 + 2\frac{12.5}{25} = 2$$

Example 2.12: A stepped shaft stepped down from 50mm diameter to 25mm diameter with a fillet radius of 5mm is subjected to an axial pull of 10kN. Determine the maximum stress induced in the member taking stress concentration into account.

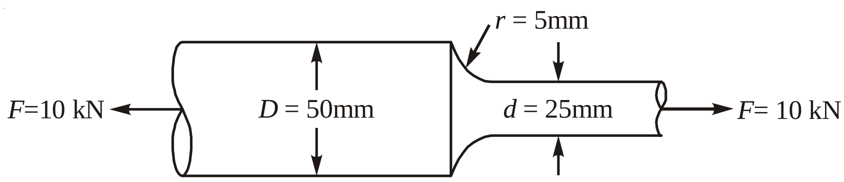


Figure-2.20

Solution:

Refer figure 4.20A/4.24 MDH

Stress concentration factor

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

Therefore, $$\sigma_{\text{nominal}} = \frac{Load}{Area}$$

$$= \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

$$= \frac{4 \times 10000}{\pi (25)^2} = 20.37\ N/mm^2$$

For the shaft under consideration,

$$\frac{D}{d} = \frac{50mm}{20mm} = 2$$

$$\frac{r}{d} = \frac{5mm}{25mm} = 0.2$$

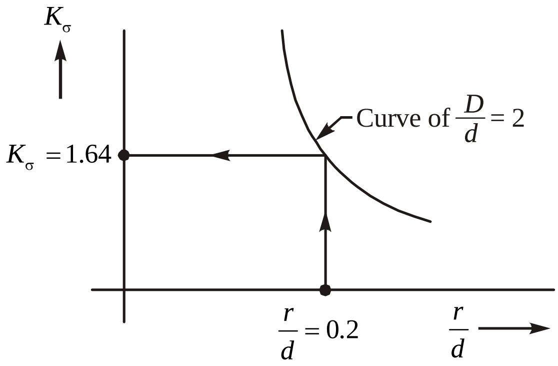


Figure-2.21

From figure-4.20 A/4.24MDH, corresponding to $\frac{D}{d} = 2$ and $\frac{r}{d} = 0.2$, stress concentration factor $K_\sigma = 1.54$.

Therefore, $\sigma_{\max} = 1.64 \times 20.37$

$= 33.4\ N/mm^2$

Example 2.13: A grooved shaft shown in figure-2.22 is subjected to a bending moment of 20 N-m. Determine the maximum stress induced in the shaft taking stress concentration into account.

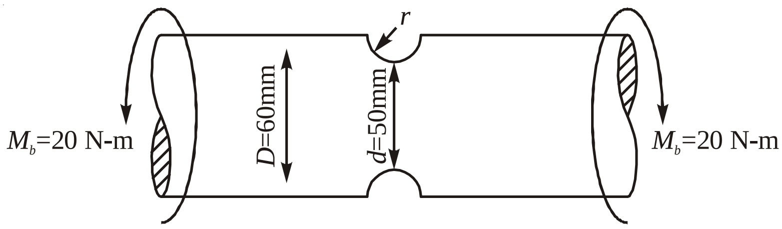


Figure-2.22

Solution:

Refer figure-2.22 (Figure 4.16A/4.20 MDH)

Stress concentration factor

$$K_\sigma = \frac{\sigma_{b\,\max}}{\sigma_{b\,\text{nominal}}}$$

Therefore,

$$\sigma_b = \text{Maximum bending stress induced in the member} = K_\sigma \sigma_{b\text{nominal}}$$

To find K_σ, the stress concentration factor

For the given shaft,

$$\frac{D}{d} = \frac{60mm}{50mm} = 1.2$$

$$r = \frac{60-50}{2} = 5mm; \qquad \frac{r}{d} = \frac{5}{50} = 0.1$$

Corresponding to $\frac{D}{d} = 1.2$ and $\frac{r}{d} = 0.1$, the stress concentration factor $K_\tau = 1.91$

To find $\sigma_{b\text{nominal}}$

$$\sigma_{b\text{nominal}} = \frac{M_b}{Z_b}$$

$$= \frac{M_b}{\frac{\pi d^3}{32}} = \frac{32M_b}{\pi d^3}$$

$$= \frac{32 \times 20 \times 10^3}{\pi (50)^3} = 1.63\, N/mm^2$$

$$\sigma_{b\max} = 1.91 \times 1.63 = 3.113\, N/mm^2$$

Therefore, *The maximum bending stress induced in the member* $= 3.113\, N/mm^2$.

Example 2.14: A notched flat plate shown in figure-2.23 is subjected to a pull of 5kN. Determine the maximum stress induced in the member taking stress concentration into account.

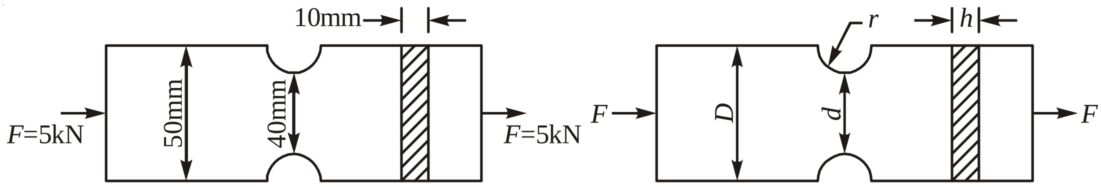


Figure-2.23

Solution:

The plate is subjected to tensile load $F = 5kN$

The stress concentration factor for axial load is given by

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

Therefore, $\sigma_{\max} = K_\sigma \sigma_{nominal}$

Where, $\sigma_{nominal} = \frac{F}{A} = \frac{F}{hd}$

$$= \frac{5\times10^3}{10\times40} = 12.5\, N/mm^2$$

For the plate under consideration, referring figure-2.23 (MDH 2.22A/4.28)

$$D = 50mm$$

$$d = 40mm$$

$$r = \left(\frac{D-d}{2}\right) = \frac{50-40}{2} = 5mm$$

Hence, $\frac{D}{d} = \frac{50}{40} = 1.25$

$$\frac{r}{d} = \frac{5}{40} = 0.125$$

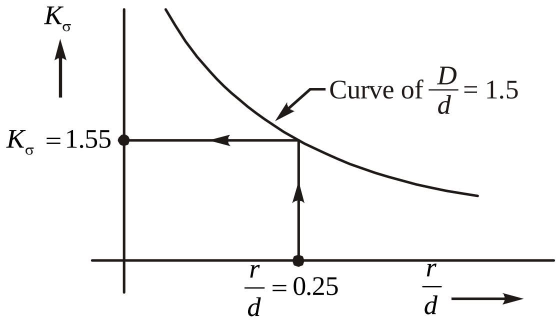


Figure-2.24

Corresponding to $\frac{D}{d} = 1.25$ and $\frac{r}{d} = 0.125$, $K_\sigma = 2.25$

$$\sigma_{\max} = 2.25 \times 12.5 = 28.125\, N/mm^2$$

Therefore, The maximum stress induced in plate $= 28.125\, N/mm^2$.

Example 2.15: A notched flat plate shown in figure-2.25 is subjected to a bending moment of 10 N-m. Determine the maximum stress induced in the member taking stress concentration into account.

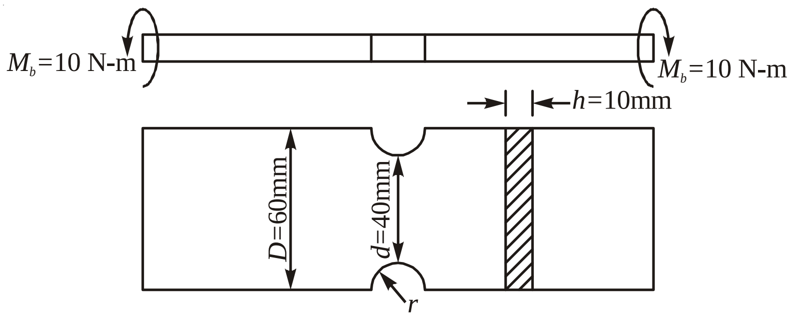


Figure-2.25

Solution:

For bending loads, stress concentration factor

$$K_\sigma = \frac{\sigma_{b\,\max}}{\sigma_{b\,\text{nominal}}}$$

Therefore, $\sigma_{b\max} = K_\sigma \sigma_{bnominal}$

To find K_σ, the stress concentration factor

For the given plate,

$$\frac{D}{d} = \frac{60mm}{40mm} = 1.5$$

$$r = \frac{D-d}{2} = \frac{60-40}{2} = 10mm$$

Therefore, $\frac{r}{d} = \frac{10mm}{40mm} = 0.25$

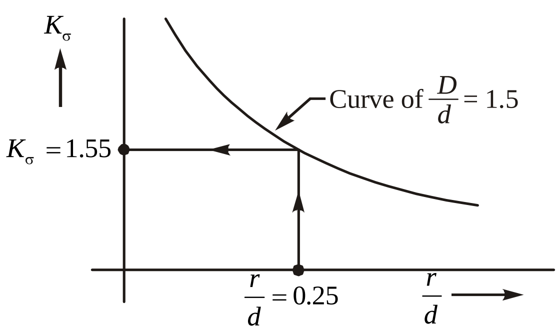


Figure-2.26 (Figure 4.23A/4.29MDH)

Therefore, referring figure2.26 (Figure 4.23A/4.29MDH), corresponding to $\frac{D}{d} = 1.5$ and $\frac{r}{d} = 0.25,\ K_\sigma = 1.55$

To find $\sigma_{b\text{nominal}}$

$$\sigma_{b\text{nominal}} = \frac{M_b}{Z_b}$$

$$= \frac{M_b}{\frac{hd^2}{6}} = \frac{6M_b}{hd^2};\left[Z_b = \frac{I}{c} = \frac{hd^2}{6}\ \text{for rectangular plate}\right]$$

$$= \frac{6\times10\times10^3}{10(40)^2} = 3.75\,N/mm^2$$

Therefore, $\sigma_{b\max} = 1.55\times3.75 = 5.8125\,N/mm^2$

Therefore, *The maximum bending stress induced in the member* $= 5.8125\,N/mm^2$.

Example 2.16: A filleted flat plate shown in figure-2.27 is subjected to a pull of 5kN. Determine the maximum stress induced in the member taking stress concentration into account.

Solution:

Stress concentration factor for member loaded in tension is

$$K_\sigma = \frac{\sigma_{max}}{\sigma_{nominal}}$$

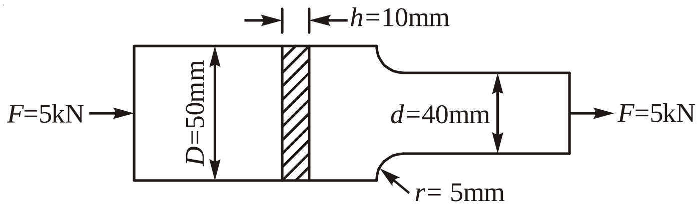


Figure-2.27

To find K_σ

For plate under consideration,

$D = 50mm,\quad d = 40mm,\quad r = 5mm,\quad h = 10mm$

$$\frac{D}{d} = \frac{50}{40} = 1.25$$

$$\frac{r}{d} = \frac{5}{40} = 0.125$$

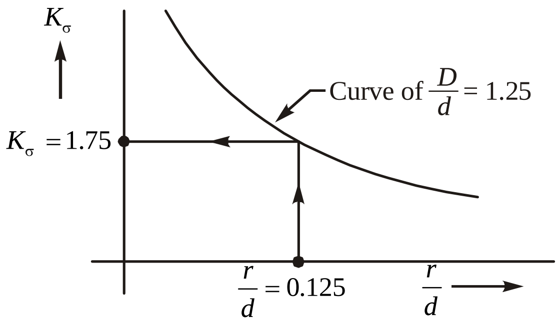


Figure-2.28 (4.24A MDH)

Corresponding to $\frac{D}{d} = 1.25$ and $\frac{r}{d} = 0.125$, $K_\sigma = 1.75$ (referring figure 4.24A/4.30)

$$\sigma_{max} = 1.75 \times 12.5 = 21.875\, N/mm^2$$

Therefore, *The maximum tensile stress induced in the member* $= 21.875\, N/mm^2$.

Example 2.17: A filleted plate shown in figure-2.29 is subjected to a bending moment of 10 N-m. Determine the maximum bending stress induced in the member taking stress concentration into account.

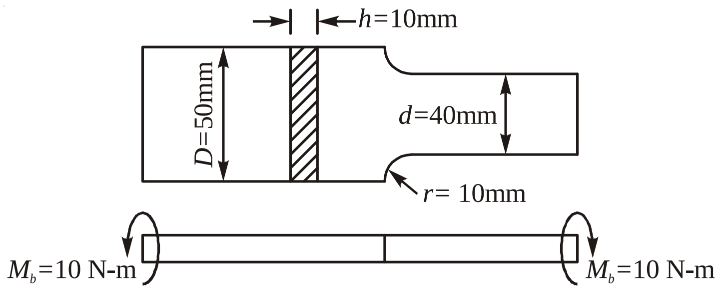


Figure-2.29

Solution:

Stress concentration factor

$$K_\sigma = \frac{\sigma_{b\,\max}}{\sigma_{b\text{ nominal}}}$$

Therefore, $\sigma_{b\max} = K_\sigma \sigma_{bnominal}$

To find K_σ,

For the given plate,

$$\frac{D}{d} = \frac{60}{40} = 1.5 \text{ and } \frac{r}{d} = \frac{10}{40} = 0.25$$

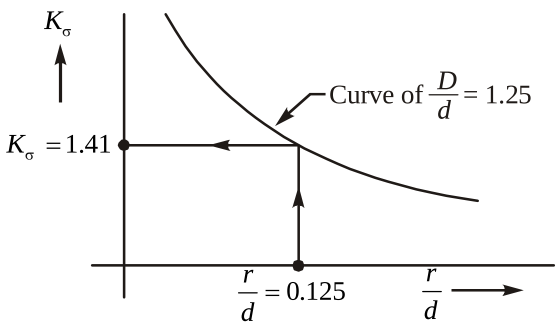


Figure-2.30 (4.25A MDH)

From figure (4.25A / 4.31 MDH), corresponding to $\frac{D}{d} = 1.5$ and $\frac{r}{d} = 0.25,$ stress concentration factor $K_\sigma = 1.41$

$$\sigma_{bnominal} = \frac{M_b}{Z_b}$$

$$= \frac{M_b}{\frac{hd^2}{6}} = \frac{6M_b}{hd^2}; Z_b = \frac{I}{c} = \frac{hd^2}{6} \text{ for rectangular plate}$$

$$\sigma_{bnominal} = \frac{6 \times 10 \times 10^3}{10(40)^2} = 3.75\, N/mm^2$$

Therefore, $\sigma_{\max} = 1.41 \times 3.75 = 5.2875\, N/mm^2$

Therefore, *The maximum tensile stress induced in the plate* $= 5.2875\, N/mm^2$.

Example 2.18: A bar of rectangular section is subjected to an axial pull of 500kN. Calculate its thickness if the allowable tensile stress in the bar is 200MPa.

(VTU, February 2002)

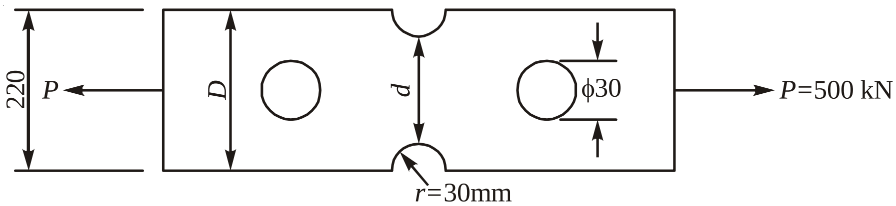
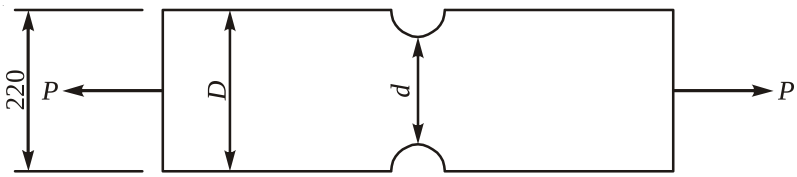


Figure-2.31

Solution:

i) Considering only groove



Stress concentration factor,

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}}$$

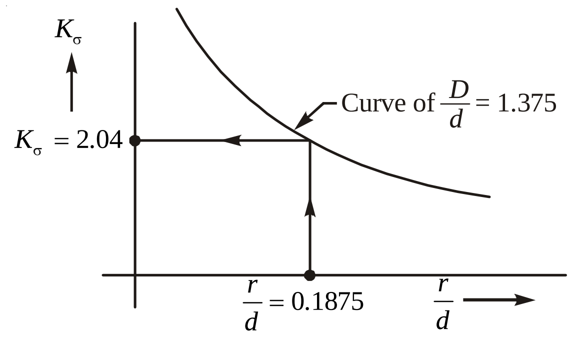
For the given plate,

$$d = D - 2r$$

$$= 220 - 2 \times 30 = 160mm$$

$$\frac{D}{d} = \frac{220}{220 - (2 \times 30)} = 1.375$$

And $\frac{r}{d} = \frac{30}{160} = 0.1875$



Corresponding to $\frac{D}{d} = 1.375$ and $\frac{r}{d} = 0.1875$, $K_\sigma = 2.04$ (Figure 4.22A/4.28 MDH)

From, $\sigma_{\max} = K_\sigma \sigma_{nominal}$

$$\sigma_{nominal} = \frac{\sigma_{\max}}{K_\sigma}$$

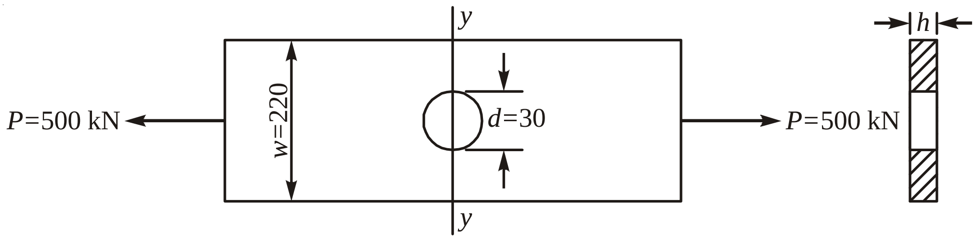
$$= \frac{200}{2.04} = 98.04\, N/mm^2$$

But, $\sigma_{nominal} = \frac{P}{dh}$

$$= \frac{500 \times 10^3}{160 \times h} = 98.04$$

Therefore, $h = 31.875mm$

ii) Considering section at hole



For the given plate,

$$\frac{d}{w} = \frac{30}{220} = 0.1364$$

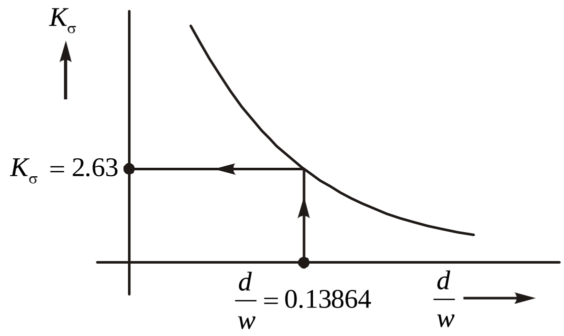


Figure-2.32 (Figure 4.5 MDH)

Corresponding to $\frac{d}{w} = 0.1364,$ stress concentration factor $K_\sigma = 2.63$ (Refer figure 4.5 MDH)

Now, $$K_\sigma = \frac{\sigma_{\max}}{\sigma_{nominal}}$$

Therefore, $$\sigma_{nominal} = \frac{\sigma_{\max}}{K_\sigma}$$

$$= \frac{200}{2.63} = 76.05\, N/mm^2$$

But, $$\sigma_{nominal} = \frac{p}{(w-d)h}$$

$$= \frac{500 \times 10^3}{(220-30)h} = 76.05$$

Therefore, $h = 34.6mm$

Adopt $h = 34.6mm,$ (higher of the two values of h)

Example 2.19: Determine the maximum stress induced in following cases taking stress concentration into account.

i) ***A rectangular plate of size*** $50mm \times 80mm$ ***with a hole of 10mm diameter drilled at the centre is loaded in axial tension of 10kN. The thickness of plate is 10mm.***

ii) ***A circular shaft of 45mm diameter stepped down to 30mm diameter having a fillet radius of 6mm subjected to a twisting moment of 150N-m.***

(VTU, July/August 2003).

Solution:

Case (i)

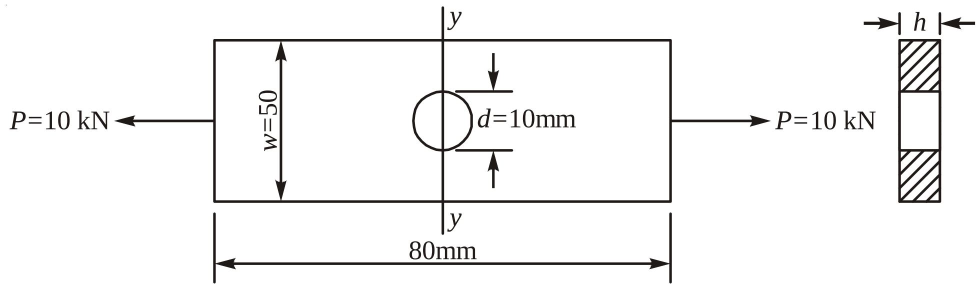
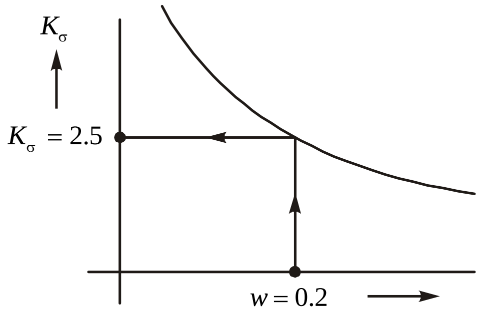


Figure-2.33

For the given plate, $\frac{d}{w} = \frac{10}{50} = 0.2$



Referring to figure-4.5/4.8 MDH for $\frac{d}{w} = 0.2,$ stress concentration factor $K_\sigma = 2.5$

Stress concentration factor (in tension)

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{nominal}}$$

Therefore, $\sigma_{\max} = K_\sigma \times \sigma_{nominal}$

Where, $\sigma_{nominal} = \frac{p}{(w-d)h}$

$$= \frac{10 \times 10^3}{(50-10)10} = 25\, N/mm^2$$

$$\sigma_{\max} = 2.5 \times 25 = 62.5\, N/mm^2$$

i.e., Maximum tensile stress in the plate taking stress concentration into account is $62.5\, N/mm^2$

Case (ii)

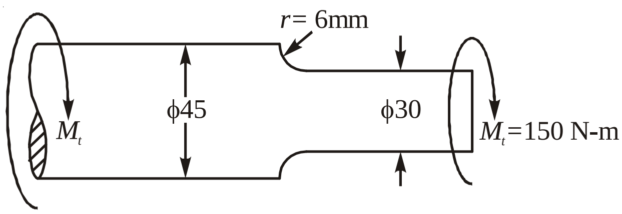


Figure-2.34

For the given stepped shaft

$$D = 45mm, \qquad r = 6mm, \qquad d = 30mm$$

Therefore, $\quad \dfrac{D}{d} = \dfrac{45}{30} = 1.5$

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

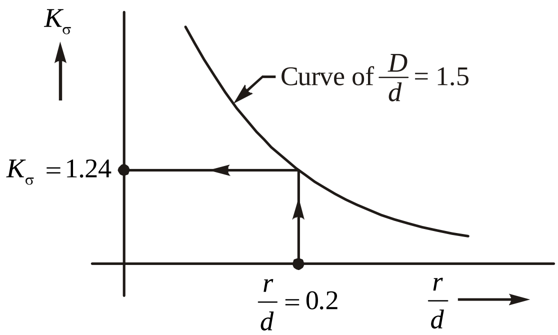


Figure-4.19A MDH

Referring to figure-4.19A/4.23 MDH for $\dfrac{D}{d} = 1.5$ and $\dfrac{r}{d} = 0.2$ stress concentration factor $K_\tau = 1.24$

For torsion, stress concentration factor K_τ is given by,

$$K_\tau = \frac{\tau_{\max}}{\tau_{nominal}}$$

Where,

Therefore, $\sigma_{\max} = K_\sigma \times \sigma_{nominal}$

$$\tau_{nominal} = \frac{M_t}{Z_t} = \frac{16M_t}{\pi d^3}$$

$$\tau_{nominal} = \frac{16\left(150\times10^3\right)}{\pi\left(30\right)^3} = 28.2942\,N/mm^2$$

Therefore, *Maximum shear stress induced in the shaft takes stress concentration into account is*

$$\tau_{\max} = K_\tau \tau_{nominal}$$
$$= 1.24 \times 28.2942$$
$$= 35.085\,N/mm^2$$

Example 2.20: Determine the maximum stress induced in the following cases taking stress concentration into account

i) A ***rectangular plate of width = 60mm, thickness = 10mm together with a hole of 10mm diameter in the centre under an axial load of 10kN.***

ii) A stepped shaft having a maximum diameter 50mm and minimum diameter 25mm with a fillet radius 6mm under a bending moment of 50N-m.

iii) The same stepped shaft under a twisting moment of 50N-m.

(VTU, July/August 2004)

Solution:

i)

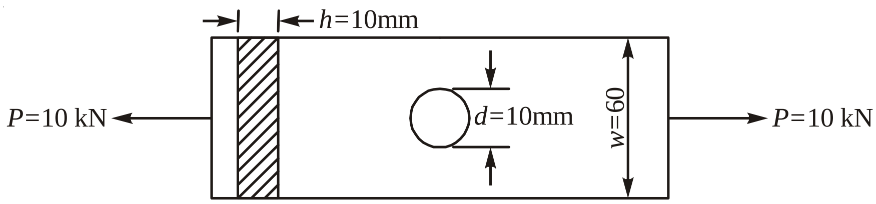


Figure-2.35: Flat plate, with hole subjected to tensile load

For tensile load,
Stress concentration factor

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_{nominal}}$$

Therefore, $\sigma_{\max} = K_\sigma \sigma_{nominal}$

Where, $\sigma_{nominal} = \dfrac{Load}{Minimum\ area\ of\ cross\ section} = \dfrac{p}{(w-d)h}$

Where, $w = width\ of\ plate = 60mm$

$d = diameter\ of\ hole = 10mm$

$h = thickness\ of\ plate = 10mm$

$$\sigma_{nominal} = \frac{10\times10^3}{(60-10)10} = 20\,N/mm^2$$

To find K_σ

For the given plate, $\dfrac{d}{w} = \dfrac{10}{60} = 0.167$ corresponding to $\dfrac{d}{w} = 0.167, K_\sigma = 2.55$ (from figure 4.5/4.8 MDH)

Therefore, $\tau_{max} = 2.55\times20 = 51\,N/mm^2$

ii) Bending moment

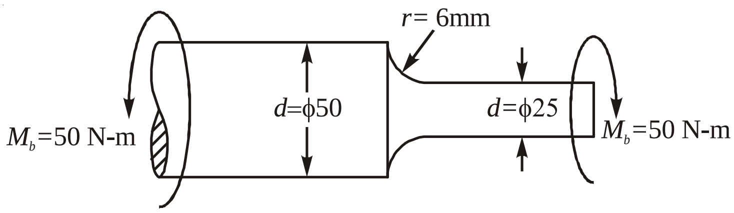


Figure-2.36: Stepped shaft subjected to bending moment

Stepped shaft subjected to bending moment

For bending loads, the stress concentration factor K_σ is given by,

$$K_\sigma = \frac{\sigma_{b\max}}{\sigma_{bnominal}}$$

Therefore, $\sigma_{b\max} = K_\sigma \times \sigma_{bnominal}$

Where, $\sigma_{bnominal} = \dfrac{M_b}{Z_{b\min}}$

$M_b = 50\text{N-m} = 50\times10^3\,N\text{-}mm$

$$Z_b = \frac{\pi d^3}{32} = \frac{\pi}{32} \times (25)^3 = 1533.98\, mm^3$$

Therefore, $\sigma_{bnominal} = \dfrac{50 \times 10^3}{1533.98} = 32.595\, N/mm^2$

To find K_σ

For the stepped shaft under study,

$$\frac{D}{d} = \frac{50}{25} = 2; \quad \frac{r}{d} = \frac{6}{25} = 0.24$$

Corresponding to $\dfrac{D}{d} = 2$ and $\dfrac{r}{d} = 0.24, K_\sigma = 1.35$ (From figure 4.21A/4.26 MDH)

Therefore, $\sigma_{b\max} = 1.36 \times 32.595 = 44.33\, N/mm^2$

The maximum bending stress in shaft $= 44.33\, N/mm^2$

iii) Torsional moment

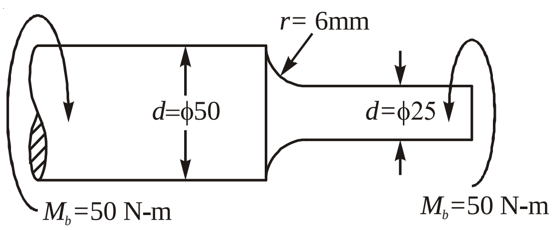


Figure-2.37: Stepped shaft subjected to torsional moment

For torsional loads,
Stress concentration factor

$$K_\tau = \frac{\tau_{\max}}{\tau_{nominal}}$$

Therefore, $\tau_{\max} = K_\tau \tau_{nominal}$

Where, $\tau_{\max} = \dfrac{M_t}{Z_{t\min}}$

$$= \frac{M_t}{\frac{\pi d^3}{16}} = \frac{16 M_t}{\pi d^3}$$

$$= \frac{16\left(50 \times 10^3\right)}{\pi \left(25\right)^3} = 16.297\, N/mm^2$$

To find K_τ

For the given shaft,

$$\frac{D}{d} = \frac{50}{25} = 2, \; \frac{r}{d} = \frac{6}{25} = 0.167$$

Corresponding to $\frac{D}{d} = 2,$ and $\frac{r}{d} = 0.167,$ $K_\tau = 1.3$

The maximum shear stress induced in shaft is

$$\tau_{\max} = K_\tau \tau_{nominal}$$
$$= 1.3 \times 16.297$$
$$= 21.186\, N/mm^2$$

Example 2.21: A round rod of diameter 1.2d has a semi circular groove of radius 0.1d. Determine the safe value for d to sustain a bending moment of 2kN-m. Select a suitable material and a factor of safety. (VTU, July/August 2005)

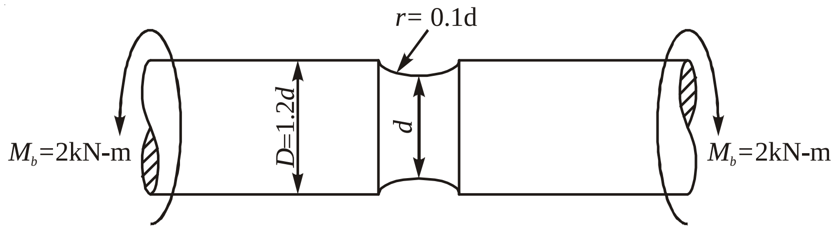


Figure-2.38

Solution:

For the given shaft,

$$\frac{D}{d} = \frac{1.2d}{d} = 1.2$$

$$\frac{r}{d} = \frac{0.1d}{d} = 0.1$$

Corresponding to $\frac{D}{d}=1.2$ and $\frac{r}{d}=0.1,$ we have stress concentration factor $K_\sigma=1.88$ (Figure 4.16A)

Now, $K_\sigma=\frac{\sigma_{b\max}}{\sigma_{bnominal}}$ for bending loads.

$$\sigma_{b\max}=K_\sigma\sigma_{bnominal}$$

Where, $\sigma_{bnominal}=\frac{M_b}{Z_{b\min}}$

$$=\frac{M_b}{\frac{\pi d^3}{32}}=\frac{32M_b}{\pi d^3}$$

$$M_b=2kN\text{-m}$$

$$=\left(2\times10^3\right)10^3\text{ N-mm}$$

$$\sigma_{bnominal}=\frac{32\left(2\times10^3\times10^3\right)}{\pi d^3}=\frac{20.37\times10^6}{d^3}$$

Assuming 40C8 as material of shaft, $\sigma_{ut}=600MPa\left(average\ value\right)$

$$\sigma_y=0.6\sigma_{ut}$$

$$=360MPa$$

$$\sigma_b=\frac{\sigma_y}{f.o.s}$$

$$=\frac{360}{3}=120$$

Therefore, $120=\frac{1.88\times20.37\times10^6}{d^3}$

Therefore, $d=68.34mm$

$$D=1.2\times68.34=82mm$$

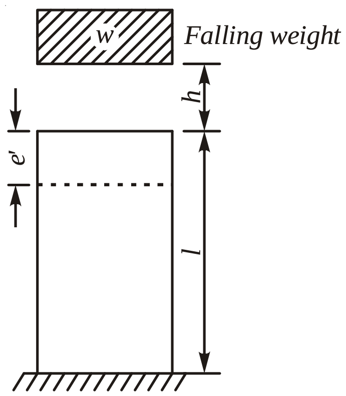
$$r=0.1d=0.1\times68.34=6.834mm$$

Impact Strength

If a moving body strikes another body, the second body is subjected to an impact which is equal to the kinetic energy of moving body. A practical example of impact would be the head on collision of two vehicles. The stress induced due to impact load is called impact stress.

Derivation of expression for Impact stress

Consider a weight '*W*' to fall through a height '*h*' striking axially, the free end of a rod of length '*l*' and area of cross section '*A*'.



Let e' be the deformation caused due to impact. The external work done by the weight or potential energy of falling weight

$$= W\left(h + e'\right) \qquad \text{... (1)}$$

The strain energy absorbed by the member is

$$U = \left(\frac{F'}{2}\right)e' \qquad \text{... (2)}$$

Where, $\frac{F'}{2}$ is the mean force on member.

Equating equations (1) and (2)

$$W\left(h + e'\right) = \frac{F'}{2}e'$$

Where, $F' = \sigma' A$; $\sigma' = \text{stress due to impact}$

Therefore, $W\left(h + e'\right) = \frac{\sigma' A}{2}e'$

Note: Here, $\frac{w}{A} = \sigma = static\ stress = stress\ due\ to\ static\ load$

Therefore, $\sigma' = 2\sigma\left[\frac{h+e'}{e'}\right]$

The Impact strain, $\in' = \frac{Deformation}{Original\ length}$

$$= \frac{e'}{l}\ But\ \in' = \frac{\sigma'}{E}$$

i.e., $\frac{\sigma'}{E} = \frac{e'}{l}\ or\ e' = \frac{\sigma' l}{E}$

Substituting the value of e' in equation (3)

$$\sigma' = 2\sigma\left[\frac{h+\frac{\sigma' l}{E}}{\frac{\sigma' l}{E}}\right]$$

$$= 2\sigma\left[\frac{Eh+\sigma' l}{\sigma' l}\right]$$

$$= \frac{2\sigma Eh}{\sigma' l} + 2\sigma$$

$$\sigma' = \frac{2\sigma Eh + 2\sigma\sigma' l}{\sigma' l}$$

i.e., $(\sigma')^2 = \frac{2\sigma Eh}{l} + 2\sigma\sigma'$

i.e., $(\sigma')^2 - \sigma'(2\sigma) - \frac{2\sigma Eh+}{l} = 0$

Solving the above Quadratic equation, we get

$$\sigma' = \frac{2\sigma \pm \sqrt{(2\sigma)^2 - (4)(1)\left[-\frac{2\sigma Eh}{l}\right]}}{2\times 1}$$

$$\sigma' = \frac{2\sigma \pm \sqrt{4\sigma + \frac{8\sigma Eh}{l}}}{2}$$

$$= \sigma \pm \sqrt{\sigma^2 + \frac{2\sigma Eh}{l}}$$

The positive root of the equation (in other words the maximum value of impact stress) is given by

$$\sigma' = \sigma + \sqrt{\sigma^2 + \frac{2\sigma Eh}{l}}$$

$$= \sigma\left[1 + \sqrt{1 + \frac{2Eh}{l\sigma}}\right]$$

Substituting $\sigma = \frac{w}{A}$ in the above equation,

$$\sigma' = \frac{w}{A} + \sqrt{\left(\frac{w}{A}\right)^2 + 2\frac{w}{A}\frac{Eh}{l}}$$

$$= \frac{w}{A} + \sqrt{\left(\frac{w}{A}\right)^2 + 2\frac{w}{A}\left(\frac{\frac{w}{A}}{\frac{w}{A}}\right)\frac{Eh}{l}}$$

$$\sigma' = \frac{w}{A} + \left[1 + \sqrt{1 + 2\frac{A}{w} \times \frac{Eh}{l}}\right] \qquad \text{... E(3.12)}$$

Since the strains are directly proportional to stresses,

$$e' = e\left[1 + \sqrt{1 + \frac{2h}{e}}\right] \qquad \text{... E(3.13)}$$

For the case of bending stress due to impact

$$\sigma_b' = \sigma_b\left[1 + \sqrt{1 + \frac{2h}{y}}\right]$$

Where, *y = Deflection due to static load.*

σ_b = Bending stress due to impact load.

Impact Factor

The ratio of maximum stress to the static stress is called Impact factor.

For axial loads, Impact factor $= \dfrac{\sigma'}{\sigma}$

$$= 1 + \sqrt{1 + \frac{2hAE}{wl}} \quad \text{... from equation (3.16)}$$

For bending loads,

Impact factor $= \dfrac{\sigma_b'}{\sigma_b}$

$$= 1 + \sqrt{1 + \frac{2h}{y}} \quad \text{... from equation (3.16)}$$

Note: If the weight 'W' is applied suddenly but it does not have an appreciable velocity, the value 'h' to be substituted is zero. The from equation (3.12)

$$\sigma' = \frac{w}{A}\left[1 + \sqrt{1}\right] = \frac{2w}{A} = 2\sigma$$

Similarly from equation (3.16),

$$\sigma_b' = \sigma_b\left[1 + \sqrt{1}\right] = 2\sigma_b$$

That is the stress produced by impact load is twice that produced by static load and the deformation due to impact is twice due to static load.

Example 2.22: A 500N weight dropped through a height of 25mm impacts the centre of a 300mm long simply supported steel square beam. If the maximum allowable bending stress is 90MPa, find the diameter of the beam and the maximum deflection. Neglect the inertia effect. Take E = 200GPa.

Solution:

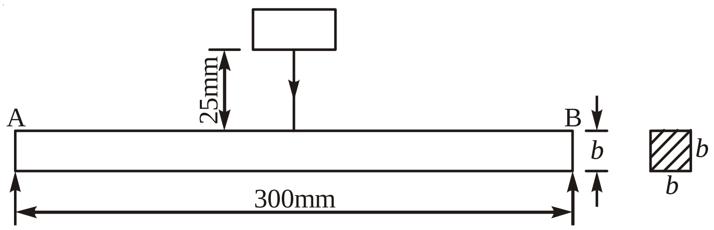


Figure-2.39

Maximum bending moment

$$M_b = \frac{wl}{4}$$

$$= \frac{500 \times 300}{4} = 37500 \text{ N-mm}$$

Bending stress σ_b due to M_b is

$$\sigma_b = \frac{M_b}{Z_b} \quad where \quad Z_b = \frac{I}{c}$$

For beam of rectangular section,

$$I = \frac{bh^3}{12} \text{ and } c = \frac{h}{2}$$

Therefore, $$Z_b = \frac{\frac{bh^3}{12}}{\frac{h}{2}} = \frac{bh^2}{6}$$

For a square section, $b = h$

Therefore, $$Z_b = \frac{b^3}{6}$$

And $$I = Z_b c$$

$$= \frac{b^3}{6} \times \frac{b}{2} = \frac{b^4}{12}$$

Therefore, $$\sigma_b = \frac{M_b}{Z_b} = \frac{37500}{b^3} \times 6$$

Static deflection due to load W is,

$$y = \frac{wL^3}{48EI} \qquad \text{... T(2.7)}$$

$$= \frac{500 \times (300)^3}{48 \times 200 \times 10^3 \times \frac{b^4}{12}} = \frac{16875}{b^4}$$

Impact stress due to bending is given by,

$$\sigma' = \sigma_b = \left[1 + \sqrt{\frac{1+2h}{y}}\right] \qquad \text{... E(3.16)}$$

Where, $\sigma_b = \frac{M_b}{Z_b} = \frac{37500 \times 6}{b^3} = 25\, N/mm^2$

$$h = Height\ of\ \ fall = 25mm$$

$$y = Static\ deflection\ of\ beam = \frac{16875}{b^4}$$

$$\sigma_b' = Maximum\ allowable\ bending\ stress = 90MPa$$

Therefore, $90 = \frac{37500 \times 6}{b^3}\left[1 + \sqrt{1 + \frac{2 \times 25 \times b^4}{16875}}\right]$

$$\frac{90h^3}{225000} = 1 + \sqrt{1 + 2.96296 \times 10^{-3} b^4}$$

i.e., $\frac{90h^3}{225000} - 1 = \sqrt{1 + \frac{2.96296b^4}{10^3}}$

By trail and error, width $b = 136mm.$

Therefore, the dimensions of cross section of beam

$$= 136mm \times 136mm\left(square\ section\right)$$

To find the maximum deflection $\left(y_{\max} = y'\right)$

Static deflection, $y = \frac{16875}{b^4}$ from equation (i)

$$= \frac{16875}{(136)^4} = 4.9327 \times 10^{-5}$$

$$y' = y\left[1 + \sqrt{1 + \frac{2h}{y}}\right]$$

$$= 4.9327 \times 10^{-5}\left[1 + \sqrt{1 + \frac{2 \times 25}{4.9327 \times 10^{-5}}}\right]$$

Therefore, $$y' = 4.9327 \times 10^{-5}\left[1 + \sqrt{1 + \frac{50}{4.9327 \times 10^{-5}}}\right]$$

$$y' = 0.04971mm = y_{\max}$$

Example 2.23: A simply supported beam has a concentrated point load at its centre. The load fluctuates from P to 4P. The span of beam is 500mm and is of circular cross section with a diameter of 60mm. The yield stress is 390MPa and endurance limit is 260MPa. Surface correction factor is 0.85 and factor of safety is 1.5. Calculate the value of P that can be applied.

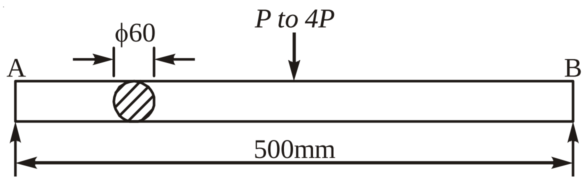


Figure-2.40

Solution:

$$M_{b\max} = \frac{w_{\max} L}{4}$$

$$= \frac{4P \times L}{4}$$

Therefore, $$PL = P \times 500 \text{ N-mm}$$

$$M_{b\min} = \frac{W_{\min} L}{4}$$

$$= \frac{PL}{4} = \frac{P \times 500}{4} = 125P \text{ N-mm}$$

Soderberg's equation for a shaft subjected to fluctuating bending moment is,

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + \frac{\sigma_{bm}}{\sigma_{yd}} = 1 \qquad \text{... E(5.52)}$$

Where, $\sigma_{ba} = \frac{M_{ba}}{Z_b}$

$$M_{ba} = \frac{M_{b\max} - M_{b\min}}{2}$$

$$= \frac{500P - 125P}{2} = 187.5P$$

$$Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$$

Therefore, $\sigma_{ba} = \frac{187.5P}{\frac{\pi d^3}{32}}$

$$= \frac{187.5P \times 32}{\pi (60)^3}$$

$$\sigma_{ba} = 8.842 \times 10^{-3} P$$

To find σ_{bm}, the mean bending stress

$$\sigma_{bm} = \frac{M_{bm}}{Z_b} \qquad \text{... E(5.46)}$$

Where, $M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$

$$= \frac{500P + 125P}{2} = 312.5P$$

Therefore, $\sigma_{bm} = \frac{3125.5P \times 32}{\pi (60)^3}$

$$\sigma_{bm} = 0.01474P$$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}} \qquad \text{... E(5.19)}$$

Where, $\sigma_{-1} = 260MPa$

$$n_{-1d} = K_{sz}\ K_{sr}\ K_l\ n_{-1a}$$

$$= 0.85 \times 0.85 \times 1 \times 1.5 = 1.08375$$

Therefore, $\sigma_{-1d} = \dfrac{260}{1.08375}$

$$\sigma_{-1d} = 239.91MPa$$

To find σ_{yd}, the design yield stress

$$\sigma_{yd} = \frac{\sigma_y}{n_{yd}} \qquad \text{... E(5.16)}$$

$$n_{yd} = K_{sz}\ K_{\sigma a}\ n_{ya}$$

$$= 0.85 \times 1 \times 1.5 = 1.275$$

Therefore, $\sigma_{yd} = \dfrac{290}{1.275} = 305.88MPa$

Substituting the values in Soderberg's equation (5.52) yields,

$$\frac{8.842 \times 10^{-3} P}{239.91} + \frac{0.01474P}{305.88} = 1$$

Therefore, $P = 11758.57\,N$

Example 2.24: A cantilever beam of width 50mm and depth 150mm is 1.2metre long and is struck by a load of 1000N which falls from a height of 20mm at its free end. Determine

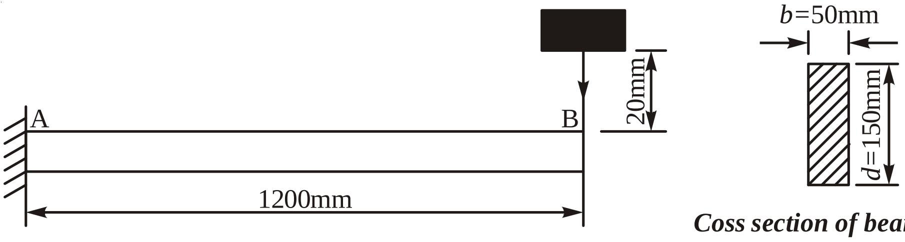
i) The maximum deflection.

ii) Maximum stress.

iii) Impact factor

iv) Maximum load Intensity. Use E = 200GPa.

Solution:



Coss section of beam AB

Figure-2.41

Impact stress due to bending is given by,

$$\sigma_b' = \sigma_b \left(1 + \sqrt{1 + \frac{2h}{y}}\right) \quad \text{... E(3.16)}$$

Where, $\sigma_b = \textit{Bending stress due to static load} = \frac{M_b}{Z_b}$

Where, $M_b = wl$

$$= 1000 \times 1200 = 12 \times 10^5 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6}$$

Where, $b = width = 50mm \; and \; d = depth = 150mm$

Therefore, $Z_b = \frac{50 \times 150^2}{6} = 187500 \; mm^3$

And $\sigma_b = \frac{12 \times 10^5}{187500} = 6.4 \; N/mm^2$

$h = 20mm$

$y =$ *Deflection due to static load for a cantilever with point load 'w'*

$$= \frac{Wl^3}{3EI}$$

$$= \frac{1000\times(1200)^3}{3\times 200\times 10^3\times 14062500} = 0.2048mm$$

$$I = \frac{bd^3}{12}$$

$$= \frac{50\times(150)^3}{12} = 14062500\ mm^4$$

Substituting we get,

$$\sigma_b' = 6.4\left(1+\sqrt{1+\frac{2\times 20}{0.2048}}\right)$$

$$\sigma_b' = 96.07\ N/mm^2$$

Therefore, Maximum bending stress in beam $= \sigma_b' = 96.07\ N/mm^2$

Maximum deflection due to impact is given by,

$$y' = y\left(1+\sqrt{1+\frac{2h}{y}}\right)$$

$$= 0.2048\left[1+\sqrt{\frac{2\times 20}{0.2048}}\right]$$

Therefore, $\quad y' = y_{\max} = 14.22\ mm$

Impact factor $\quad = \frac{\sigma_b'}{\sigma_b} = \frac{96.07}{6.4} = 15.01$

Maximum load intensity $= w\times Impact\ \ factor$

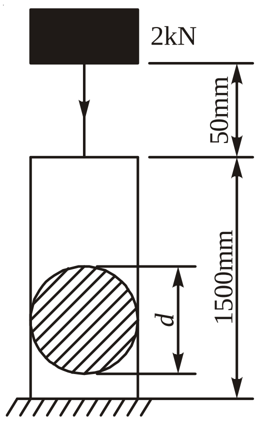
$= 1000\times 15.01$

$= 15010\,N$

Example 2.25: A steel rod 1.5metre long resists an impact load of 2kN dropped through a distance of 50mm along its axis. Limiting the maximum stress in the rod to 150MPa, determine,

i) The diameter of rod required.

ii) Impact factor. Use E = 200GPa.



Solution:

The maximum stress in the rod due to impact is given by,

$$\sigma' = \sigma\left[1+\sqrt{1+\frac{2h}{y}}\right] \quad \text{... E(3.11)}$$

Where, $\sigma' = \text{Maximum stress due to impact} = 150 MPa.$

$\sigma = \text{Axial stress due to static load}$

$$\frac{w}{A} = \frac{2\times10^3}{\frac{\pi}{4}\times d^2}$$

$$= \frac{2546.48}{d^2} N/mm^2$$

$h = \text{Height of fall} = 50mm$

$y = \text{Deflection due to static load}$

$$\frac{wL}{AE} = \frac{2\times10^3\times1500}{\frac{\pi}{4}d^2\times200\times10^3}$$

$$= \frac{190.98}{d^2} mm$$

Substituting in equation (3.11) we get,

$$150 = \frac{2546.48}{d^2}\left[1 + \sqrt{1 + \frac{2 \times 50}{\frac{19.098}{d^2}}}\right]$$

$$0.0589d^2 = 1 + \sqrt{1 + \frac{5.2362}{d^2}}$$

Therefore, $0.0589d^2 - 1 = \sqrt{1 + 5.2362d^2}$

Squaring both sides,

$$\left(0.0589d^2\right)^2 + 1 - 2 \times 0.0589d^2 = 1 + 5.2362d^2$$

$$3.4621 \times 10^{-3} d^4 - 0.1178d^2 = 5.2362d^2$$

$$3.46921 \times 10^{-3} d^4 = 5.354d^2$$

$$0.0589d^2 = 2.3139d$$

Therefore, $d = 39.28mm$

Now, Impact factor $= \frac{\sigma'}{\sigma} = \frac{150}{\sigma}$

Where, $\sigma' = 150MPa$

$$\sigma = \frac{2546.48}{d^2}$$

$$= \frac{2546.48}{(39.28)^2} = 1.6504$$

Therefore, Impact factor $= \frac{1560}{1.6504} = 90.88$

Example 2.26: A simply supported beam of length 3meters is struck at its centre by a weight 5kN. The cross section of beam is rectangular with depth of beam = 250mm and width 100mm. Limiting the maximum stress in the beam to 100MPa, determine the height of fall. Use G = 200GPa.

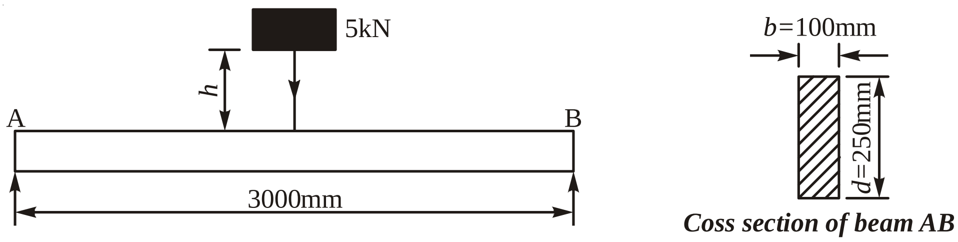


Figure-2.42

Solution:

Maximum bending stress due to impact is

$$\sigma_b' = \sigma_b \left[1 + \sqrt{1 + \frac{2h}{y}} \right] \qquad \text{... E(3.11)}$$

Where, $\sigma_b' = 100 MPa$

$$\sigma_b = \frac{M_b}{Z_b} = \textit{Bending stress due to static load}$$

For a simply supported beam of length L with point load W at the centre, Bending moment M_b is

$$M_b = \frac{WL}{4}$$

$$= \frac{(5 \times 10^3)(3000)}{4} = 375 \times 10^4 \text{ N-mm}$$

$$Z_b = \textit{Section modulus} = \frac{I}{c}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6} mm^3$$

Where, $b = 100mm$

$d = 250mm$

Therefore, $Z_b = \dfrac{100 \times (250)^2}{6} = 1041666.67\, mm^3$

And $\sigma_b = \dfrac{375\times10^4}{1041666.67} = 3.599\, N/mm^2$

$y =$ Deflection due to static load for a simply supported beam with point load W at the centre is

$$y = \frac{WL^3}{48EI}$$

$$= \frac{5\times10^3 \times 3000^3}{40\times 200\times10^3 \times I}$$

Where, $I = \dfrac{bd^3}{12}$

$$= \frac{100\times(250)^3}{12} = 130208333.3\ mm^4$$

Therefore, $y = 0.108\, mm$

Substituting in equation (3.11) we get,

$$100 = 3.599\left(1+\sqrt{1+\frac{2h}{0.108}}\right)$$

$$26.785 = \sqrt{1+\frac{2h}{0.108}}$$

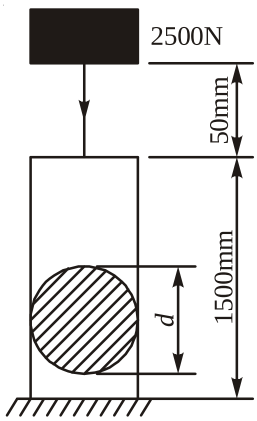
Squaring both sides, we get

$$26.785^2 = 1+\frac{2h}{0.108}$$

Therefore, $h = 38.69\, mm$

Therefore, Height from which the weight is to be dropped $= 38.69\, mm$

Example 2.27: Find the diameter of a steel rod 1.5meter long to resist an load impact of 2500N dropped through a distance of 50mm longitudinally. The maximum stress is limited to $150\, N/mm^2$. ***Use*** $E = 200\times10^3\, MPa.$



Solution:

Due to impact stress in the beam is given by,

$$\sigma' = \sigma\left[1+\sqrt{1+\frac{2h}{y}}\right] \qquad \text{... E(3.11)}$$

Where, $\sigma = Stress\ due\ to\ static\ load = 2500N$

$$= \frac{2500}{\frac{\pi d^2}{4}}$$

$$= \frac{2500\times 4}{\pi d^2} = \frac{3183.1}{d^2}$$

$$\sigma' = 150\ N/mm^2$$

$$h = Height\ of\ \ fall = 50mm$$

$$y = Deflection\ due\ to\ static\ load = \frac{WL}{AE}$$

$$= \frac{2500\times 1500}{\frac{\pi d^2}{4}\times 200\times 10^3}$$

$$= \frac{23.8732}{d^2}$$

Substituting in equation (3.11) we get,

$$150 = \frac{3183.1}{d^2}\left(1+\sqrt{1+\frac{2\times 50\times d^2}{23.8732}}\right)$$

i.e., $0.47124d^2 = 1+\sqrt{1+4.1888d^2}$

or $0.47124d^2 - 1 = \sqrt{1+4.1888d^2}$

Squaring both sides,

$$\left(0.47124d^2 - 1\right)^2 = 1+4.1888d^2$$

$$2.0207\times 10^{-3} d^4 + 1 - 0.094248d^2 = 1+4.1888d^2$$

$$2.2207\times 10^{-3} d^4 - 4.094552d^2$$

i.e., $0.0469787d^2 = 2.0235$

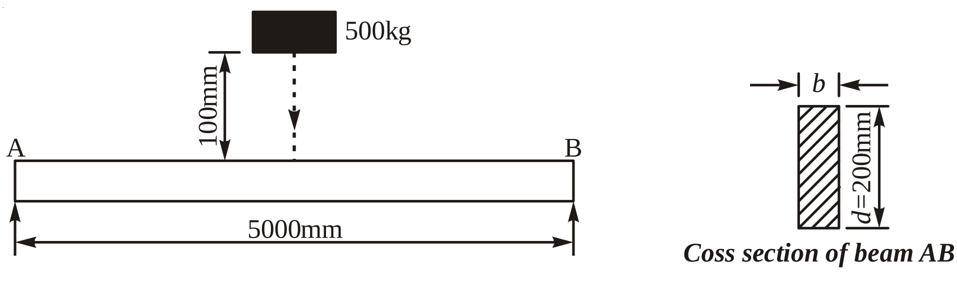
Therefore, $d = 43.0727mm$

Therefore, Diameter of rod required $= 43.0727mm$

Example 2.28: A power hammer of mass 500kg strikes the anvil supported at the mid point of a simple supported beam 5meter long. The height through which the hammer falls is 100mm. Determine the width of rectangular section beam required if the depth of section is 200mm. limit the maximum stress in the beam to 120MPa.

(VTU - Sept/Oct 2000).

Solution:



Coss section of beam AB

Figure-2.43

The beam is subjected to impact stress and the maximum value of bending stress due to impact is given by,

$$\sigma_b' = \sigma_b\left[1+\sqrt{1+\frac{2h}{y}}\right] \quad \text{... E(3.16)}$$

$$\sigma_b' = 120MPa \text{ (given)}$$

$$\sigma' = Bending\ stress\ due\ to\ static\ load = \frac{M_b}{Z_b}$$

Where, $$M_b = M_{b\max} = \frac{Wl}{4}$$

$$= \frac{(500 \times 9.81)5000}{4} = 6131250 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{bd^2}{6}$$

$$= \frac{b \times 200^2}{6} = 6666.67b\ mm^3$$

Therefore, $$\sigma_b = \frac{M_b}{Z_b}$$

$$= \frac{6131250}{6666.67b} = \frac{919.6875}{b} mm$$

$$h = Height\ of\ \ fall = 100mm$$

Deflection due to static load for a simply supported beam with point load at the centre is given by,

$$y = \frac{Wl^3}{48EI}$$

$$= \frac{500 \times 9.81 \times (5000)^3}{48 \times 200 \times 10^3 \times b\frac{(200)^3}{12}},$$

Where, $E = 200GPa = 200 \times 10^3 MPa$

$$= \frac{95.8}{b}$$

Substituting in equation (3.16) we get,

$$\sigma_b' = \frac{919.6875}{b}\left(1+\sqrt{1+\frac{2\times100\times b}{95.8}}\right)$$

$$120 = \frac{919.6875}{b}\left(1+\sqrt{1+\frac{2\times100\times b}{95.8}}\right)$$

$$0.1305b = 1+\sqrt{1+2.089b}$$

$$0.1305b-1 = \sqrt{1+2.089b}$$

Squaring both sides,

$$0.017b^2+1-0.261b = 2.089b+1$$

$$0.017b^2 = 2.35b$$

Therefore, Width of beam $b = 138.24\,mm.$

Example 2.29: A mass of 600kg falls through a height h at the midpoint of a simply supported beam of span 4.5meters. Determine the value of h such that the maximum stress induced in the beam does not exceed 160MPa. The section modulus of the section of the beam may be taken as 200000mm^3 and second moment of inertia = 1000000mm^4.

(VTU – March/April 2000)

Solution:

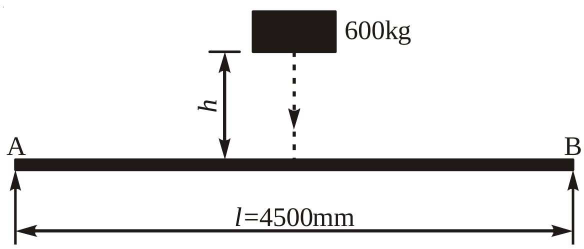


Figure-2.44

The beam is subjected to bending and hence the impact stress due to bending is given by

$$\sigma_b' = \sigma_b\left[1+\sqrt{1+\frac{2h}{y}}\right] \qquad \text{... E(3.16)}$$

Where,

σ_b' = *Maximum bending stress in beam due to impact* $= 160\,N/mm^2$ (*given*)

$$\sigma_b = Bending\ stress\ in\ beam\ due\ to\ static\ load = \frac{M_b}{Z_b}$$

$$M_b = Maximum\ bending\ moment = \frac{Wl}{4}$$

(for simply supported beam with point load at the centre)

i.e., $$M_b = \frac{600 \times 9.81 \times 4500}{4} = 6621750 \text{ N-mm}$$

$$Z_b = \frac{I}{c} = 200000$$

Therefore, $$\sigma_b = \frac{6621750}{200000} = 33.1088\, N/mm^2$$

$y = \dfrac{Wl^3}{48EI} =$ Static deflection due to load W for a simply supported beam with point load at the centre.

$$E = 200GPa = 200 \times 10^3 MPa$$

$$y = \frac{500 \times 9.81 \times (4500)^3}{48 \times 200 \times 10^3 \times 1000000} = 46.56mm$$

Therefore, $$160 = 33.1088\left(1 + \sqrt{1 + \frac{2h}{46.56}}\right)$$

$$4.8326 - 1 = \sqrt{1 + \frac{2h}{46.56}}$$

Squaring both sides and simplifying we get,

$$h = 318.68mm$$

Example 2.30: A simply supported beam of length 2meters is struck at its mid length by a load 2kN. The height of fall is 250mm. The cross section of beam is rectangular with width 100mm and depth 200mm. Determine the maximum stress in the beam when

i) Longer side of beam is vertical and the corresponding impact factor.

ii) Longer side of beam is horizontal and the corresponding impact factor.
Use E = 200GPa.

Solution:

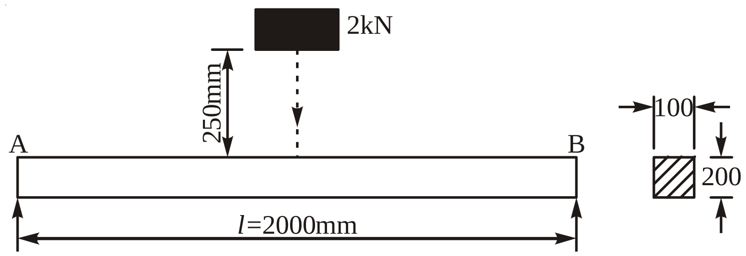


Figure-2.45

Maximum bending stress in the beam due to impact is given by,

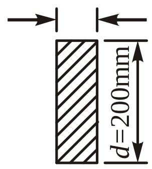
$$\sigma_b' = \sigma_b \left[1 + \sqrt{1 + \frac{2h}{y}} \right] \qquad \text{... E(3.16)}$$

Case (i) When longer edge is vertical

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $M_b = \dfrac{WL}{4}$

$$= \frac{2000 \times 2000}{4} = 10^6 \text{ N-mm}$$



(Longer side vertical)

$$Z_b = \frac{I}{c}$$

Where, $I = \dfrac{bd^3}{12}$

$$= \frac{100 \times 200^3}{12} = 66666.67\, mm^4$$

$$c = \frac{d}{2}$$

$$= \frac{200}{2} = 100\, mm$$

Therefore, $Z_b = \dfrac{66666.67}{100} = 666.67$

$$\sigma_b = \frac{1000000}{666.67} = 1500\, N/mm^2$$

$$h = 250mm$$

$$y = \textit{Deflection at the centre of beam due to static load}$$

$$= \frac{Wl^3}{48EI} \text{ (for a simply supported beam with point load at the centre)}$$

$$y = \frac{2000\times(2000)^3}{48\times 200\times 10^3\times 66666.67} = 25mm$$

Therefore, $$\sigma_b' = 1500\left(1+\sqrt{1+\frac{2\times 250}{25}}\right)$$

$$= 8373.86\, N/mm^2$$

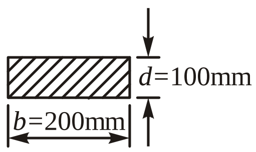
Therefore, Impact factor $= \dfrac{\sigma_b'}{\sigma_b} = \dfrac{8363.86}{1500} = 5.5826$

Case (ii) When the longer side is horizontal

$$\sigma_b = \frac{M_b}{Z_b}; \qquad \text{Where } M_b = 10^6 \text{ N-mm}$$

$$Z_b = \frac{I}{c}$$

Where, $$I = \frac{200\times 100^3}{12} = 1.67\times 10^7\ mm^4$$



(Longer side horizontal)

$$c = \frac{d}{2}$$

$$= \frac{100}{2} = 50mm$$

Therefore, $$Z_b = \frac{I}{c}$$

$$= \frac{1.67\times 10^7}{50} = 3.33\times 10^5\ mm^3$$

$$\sigma_b = \frac{10^6}{3.33 \times 10^5} = 3\, N/mm^2$$

Deflection due to static load is,

$$y = \frac{Wl^3}{48EI}$$

$$= \frac{2000 \times (2000)^3}{48 \times 200 \times 10^3 \times 1.67 \times 10^7} = 0.1 mm$$

Where, $E = 200 GPa = 200 \times 10^3 MPa$

Therefore, Maximum bending stress is,

$$\sigma_b' = \sigma_b \left[1 + \sqrt{1 + \frac{2h}{y}} \right]$$

$$= 3\left(1 + \sqrt{1 + \frac{2 \times 250}{0.1}} \right)$$

$$\sigma_b' = 215.15\, N/mm^2$$

Therefore, Impact factor $= \frac{\sigma_b'}{\sigma} = \frac{215.15}{3} = 71.72$

Example 2.31: An unknown weight falls through 20mm on to a collar rigidly attached to the lower end of a vertical bar 2metre long and 500 sq. mm section. If the maximum instantaneous extension is known to be 2mm, what is the corresponding value of unknown weight. Take E = 200GPa. (VTU – July/Aug. 2003)

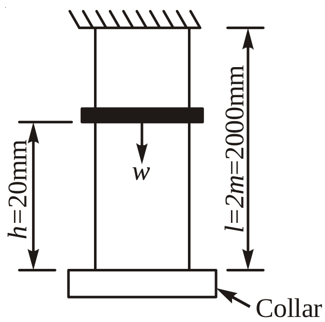


Figure-2.46

Solution:

We have,

$$Impact\ stress = \sigma' = \frac{w}{A}\left[1+\sqrt{1+\frac{2hEA}{wl}}\right] \qquad \text{... E(3.12)}$$

Where, $Impact\ stress = \sigma' = \frac{Ee'}{l}$

E = Modulus of elasticity of material $= 200GPa = 200\times10^3 MPa$

$e' = Instantaneous\ elongation = 2mm$

$l = Length\ of\ bar = 2m = 2000\,mm$

Therefore, $\sigma' = \frac{200\times10^3\times2}{2000} = 200MPa$

Substituting in equation (3.12)

$$200 = \frac{w}{500}\left[1+\sqrt{1+\frac{2\times2\times200\times10^3\times500}{w\times2000}}\right]$$

$$200 = \frac{w}{500}\left[1+\sqrt{1+\frac{2\times10^5}{w}}\right]$$

$$\frac{500\times200}{w} = 1+\sqrt{1+\frac{2\times10^5}{w}}$$

$$\frac{10^5}{w}-1 = \sqrt{1+\frac{2\times10^5}{w}}$$

Squaring both sides, we get

$$\left(\frac{10^5}{w}-1\right)^2 = 1+\frac{2\times10^5}{w}$$

$$\frac{10^{10}}{w^2}+1-\frac{2\times10^5}{w} = 1+\frac{2\times10^5}{w}$$

$$\frac{10^{10}}{w^2}-\frac{2\times10^5}{w} = \frac{2\times10^5}{w}$$

$$\frac{10^{10}}{w^2} = \frac{10^5}{w}(2+2)$$

Therefore, $w = 50000\,N$

Example 2.32: A weight of 2kN falls on the collar provided at the end of a vertical rod of length 3meter and diameter 25mm. Assuming that there is no loss of energy, determine the height through which the weight has to be dropped so as to limit the maximum stress in the bar to 200MPa. What is the elongation due to impact. Use $E = 200 \times 10^3\,MPa$

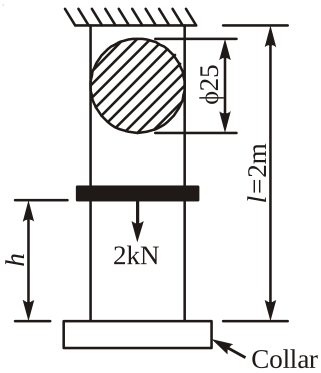


Figure-2.47

Solution

We have, $$Impact\ stress = \sigma' = \frac{w}{A}\left[1 + \sqrt{1 + \frac{2hEA}{wl}}\right] \quad \text{... E(3.12)}$$

Where, $\sigma' = 200MPa$

$w = Load = 2kN = 2 \times 10^3\ N$

$A = Area\ of\ cross\ section\ of\ bar$

$$= \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4}(25)^2 = 490.87\,mm^2$$

$h = Height\ of\ fall, mm$

$l = Length\ of\ rod = 2m = 2000\,mm$

$E = 200 \times 10^3\ N/mm^2$

Therefore, $$200 = \frac{2000}{490.87}\left(1+\sqrt{1+\frac{2\times h\times 200\times 10^3\times 490.87}{2000\times 2000}}\right)$$

$$49.087 = 1+\sqrt{1+49.087\times h}$$

$$(49.087)^2 = 1+49.087h$$

Therefore, $h = 47.087mm$

Therefore, Height of fall $= 47.087mm$

To find elongation due to impact

Elongation due to impact is

$$y' = y\left(1+\sqrt{\frac{2h}{e}}\right) \qquad \text{... E(3.13)}$$

Where, y = Deflection due to static load is

$$y = \frac{wl}{Ae}$$

$$= \frac{2000\times 2000}{490.87\times 200\times 10^3}$$

$$= 0.0407mm$$

Therefore, $$y' = 0.0407\left(1+\sqrt{\frac{2\times 47.08}{0.0407}}\right)$$

$$= 1.998mm$$

Example 2.33: A weight of 3kN falls on the flange provide at the end of a vertical steel rod of diameter d and length 3metre from a height of 50mm such that the maximum stress in the rod is 200MPa. Determine,

i) The diameter of the rod d.

ii) Deformation due to impact. Use E = 208GPa.

Solution:

We have, $$Impact\ stress = \sigma' = \frac{w}{A}\left[1+\sqrt{1+\frac{2hEA}{wl}}\right] \qquad \text{... E(3.12)}$$

Where, $\sigma' = 200MPa$

$$w = Load = 3kN = 3\times10^3\ N$$

$$h = Height\ of\ \ fall = 50mm$$

$$E = 208\,GPa = 208\times10^3\,MPa$$

$$l = Length\,of\ \ rod = 3m = 3000\,mm$$

$$A = Area\,of\ \ cross\ section\ of\ \ bar, mm^2$$

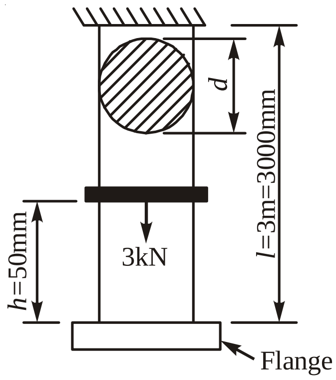


Figure-2.48

Therefore, $$200 = \frac{3\times10^3}{A}\left(1+\sqrt{1+\frac{2\times50\times208\times10^3\times A}{3\times10^3\times3000}}\right)$$

$$0.067A = \left[1+\sqrt{1+2.311A}\right]$$

$$0.067A - 1 = \left[\sqrt{1+2.311A}\right]$$

Squaring both sides,

$$\left(0.067A-1\right)^2 = 1+2.311A$$

$$4.489\times10^{-3}\times A^2 + 1 - 0.134A = 1 + 2.311A$$

Therefore, $$4.489\times10^{-3}A^2 = 2.445A$$

i.e., $$A = 544.66\ mm^2$$

But, $$A = \frac{\pi d^2}{4} = 544.66$$

Therefore, Diameter of rod, $d = 26.33mm$

To find deformation y' due to impact

$$y' = y\left(1 + \sqrt{\frac{2h}{y}}\right)$$

Where, $y = Deflection\ due\ to\ static\ load = \frac{Wl}{AE}$

$$= \frac{3 \times 10^3 \times 3000}{544.66 \times 208 \times 10^3}$$

$$= 0.07944\ mm$$

Therefore, $y' = 0.07944\left(1 + \sqrt{\frac{2 \times 50}{0.07944}}\right)$

$$y' = 2.8979\ mm$$