DESIGN FOR FATIGUE STRENGTH

Chapter 3

Introduction

In obtaining the properties such as yield strength, ultimate strength, etc., the specimen is loaded gradually and sufficient time is allowed for strain to develop in the specimen and loading is continued till fracture. Such a type of loading is called *"Static loading"*.

There are instances in which the stress vary or fluctuate between values. Consider a rotating shaft with bending loads. A particular fibre on the surface of shaft undergoes both *tension* and *compression* for each revolution of shaft. Suppose the speed of rotation of shaft is 1440 rpm, then the fibre under consideration under goes both *tension* and *compression 1440 times in one minute.* If in addition, there is *axial loading,* (due to the presence of *Helical* or *Worm gears* on the shaft) the *axial component* of stress is superimposed by *bending component* and the net result is a stress, again fluctuating. These stresses produced in the member due to fluctuating loads are called *fluctuating stress,* or *repeated* or *alternating stresses.*

A careful analysis of member failing under the action of fluctuating stresses reveals that the maximum stress at which failure occurs is far below the *ultimate strength of material* and in some cases it is even lesser than the *yield strength of material.* If *fatigue failure,* the stresses are repeated many times.

The fatigue failure begins with a *small crack.* The *initial crack* is so small that it cannot be detected by the *naked eye* and even difficult to locate using *magnaflux* or *X-ray inspection.* This crack will develop at a point of discontinuity in material such as a *change in cross section, a key way or a notch or a hole.*

Once a crack has developed, the stress concentration effect becomes greater and the crack propagates more rapidly. As the stressed area decreases in size, the stress increases in magnitude and finally the remaining area fails suddenly.

A *fatigue failure* is characterized by *two distinct areas of failure.* The first is the progressive development of crack and the second is due to the sudden fracture.

When machine member fail statically, they usually develop a very large deflection and the stress exceeds the *yield strength of material.* Thus, *static failures are visible ones* and the part can be replaced before fracture actually occurs.

Fatigue failures gives no warning; it is sudden and total and hence very dangerous.

S.N. Diagram

To determine the *strength of materials* under the action of *fatigue loads,* the specimen are subjected to *fatigue loads* of *specified magnitude* and the number of cycles of *stress reversals* are counted (recorded) till *fracture occurs.* The test is conducted on a *high speed rotating machine* in which the specimen is subjected to pure bending by means of weights. The test specimen should be carefully *machined* and *polished* with *final polishing* in an *axial direction to avoid circumferential scratches* on the *specimen.*

To determine the *fatigue strength of material, a number of tests* are necessary because of *statistical nature of fatigue. In principle, in a rotating beam test, a constant bending load is applied and the number of revolutions of the beam required for failure is recorded.*

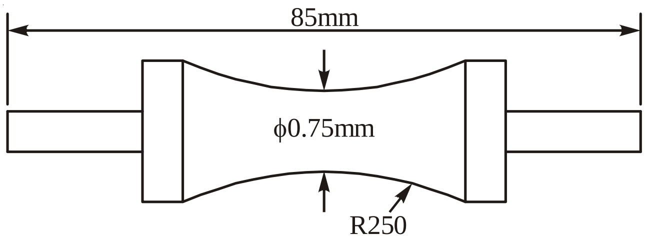

Figure-3.1: Test specimen for rotating beam fatigue testing machine

The *first test* is conducted at a stress which is *slightly lesser* than the *ultimate strength* of material and the *second test* at a stress which is *less than the stress used in first test.* The test is repeated and the results are plotted as *S-N diagram* as shown on a *semi log paper of a log-log paper.*

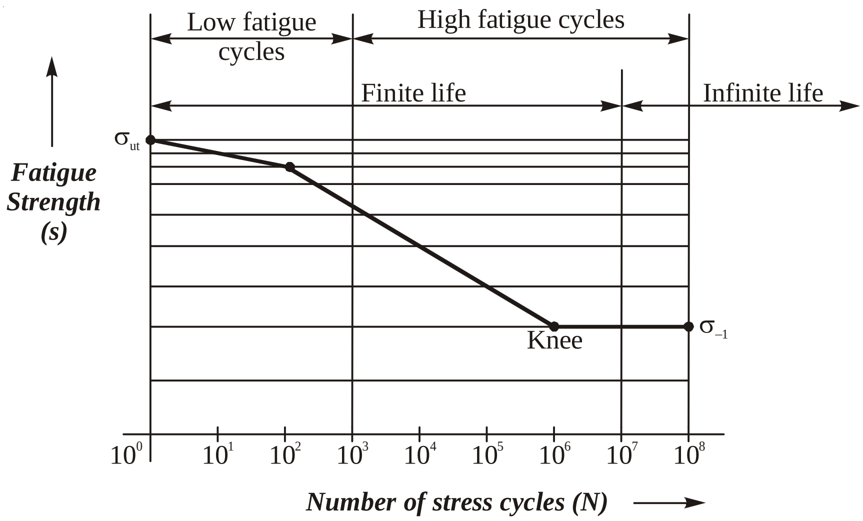

Figure-3.2

In case of Ferrous metals and alloys, the graph becomes *horizontal* after the material has been stressed for a certain *number of cycles.*

It is very important to notice that even when the *material of test specimen* and that of *mechanical element are identical,* there will be *significant changes in the two diagrams.*

In *case of steels,* a *Knee* occurs in the *S-N curve* beyond which failure will not occur, no matter *how greater the number of cycles are.* The strength corresponding to this *Knee* is called *fatigue strength or endurance strength* σ_{-1}. The *S.N. curve* never becomes *Horizontal* for *non-ferrous metals and alloys* and hence these materials *do not* have an *endurance limit.*

Fatigue failure from $N = 1$ to $N = 10$ cycles is generally classified as *low cycle fatigue.* The failures for stress cycles *greater than* 10^3 cycles are termed as *high cycle fatigue.*

In the *S.N curve* are also plotted the regions of *finite life* and *infinite life.* The *boundary* between these regions cannot be generally *defined* except for a *specific material.* For steels, the *finite life* lies upto 10^7 cycle.

Low Cycle Fatigue

In *low cycle fatigue,* studies of fatigue resistance of specimen and machine elements when they are subjected to stress reversals upto 10^3 cycles are made. A knowledge of *fatigue resistance* of *low cycle regions* is desirable for the *design of short lived devices* such as *missiles.* A *fatigue failure* begins at a *local discontinuity* such as *notch, crack, etc.* or *areas of high stress concentration.* When the *stress exceeds* the *elastic limit, plastic strain occurs.* If *fatigue fracture* is to occur, then there must exist *cyclic plastic strains.* Hence in *low cycle fatigue,* it is necessary to study the *behaviour of materials* subjected to *cyclic deformation.*

High Cycle Fatigue

In the *S.N curve,* the *region* beyond $N = 10^3$ *cycles of stress* is defined as *region of high cycle fatigue.*

Endurance Limit modifying Factors

It is unrealistic to expect that the *results* of a mechanical or a structural member to *match with the endurance strength* obtained in the laboratory by testing specimen carefully prepared and tested under closely controlled conditions. To account for these conditions a number of *modifying factors* are applied. With this we can write

$$\sigma_{-1} = K_{ST} \quad K_{sz} \quad n_a \quad K_t \quad K_f \quad K_e \quad \sigma'_{-1}$$

Where, $\sigma_{-1} = Endurance\ limit\ of\ material\ of\ member$

$\sigma'_{-1} = Endurance\ limit\ obtained\ from\ conducted\ test\ in\ the\ lab$

$K_{ST} = Surface\ factor$

$K_{sz} = Size\ factor$

$n_a = Factor\ of\ safety$

$K_t = Temperature\ factor$

$K_f = Miscellaneous\ effect\ factor$

$K_e = Modifying\ factor\ for\ stress\ concentration$

a) Surface effect

The surface of the specimen tested in laboratory is *highly polished.* Most of the machine elements do not have such a high quality finish. The surface factor depends on the *ultimate tensile strength* and the *process adopted* to produce the component such as *forging, hot rolling, cold rolled, ground* etc. this factor decreases in value with *increased tensile strength* excepting for *ground* and *polished specimens* in which the *value remains constant.*

b) Size effect

The specimen used in *rotating beam* test has a diameter of 7.5mm. The values of *endurance limit* of machine elements having larger size on different cross sections do not match with the results obtained from rotating beam method. This effect due to the *dimensions, the shape, and the method of loading* is called *size effect.* Testing of specimen having *large dimension* is very costly and requires *expensive laboratory facilities.* Because of this reason, there are only *limited data of test results.*

c) Stress concentration effects

Stress concentration in mechanical parts are due to the presence of *notches, grooves, holes, etc.,* alter the *stress distribution.* Stress concentration is to be considered for parts made of *brittle materials* or when they are subjected to *fatigue loading.*

Fatigue stress concentration factor,

$$K_f = \frac{Endurance\ limit\ of\ notch - Free\ specimen}{Endurance\ limit\ of\ notched\ specimen}$$

Modifying factor for stress concentration is given by

$$K_e = \frac{1}{K_f}$$

Notch sensitivity '*q*' is defined as

$$q = \frac{K_f - 1}{K_t - 1}$$

Or, $$K_f = 1 + q\left(K_t - 1\right)$$

Fluctuating stresses

Many times it is necessary to determine the strength of parts corresponding to stress situations other than complete reversals. The *stress fluctuate* without passing through zero.

$\sigma_{\max} = \textit{Maximum stress}$

$\sigma_{\min} = \textit{Minimum stress}$

$\sigma_r = \textit{Stress range}$

$\sigma_a = \textit{Stress amplitude}$

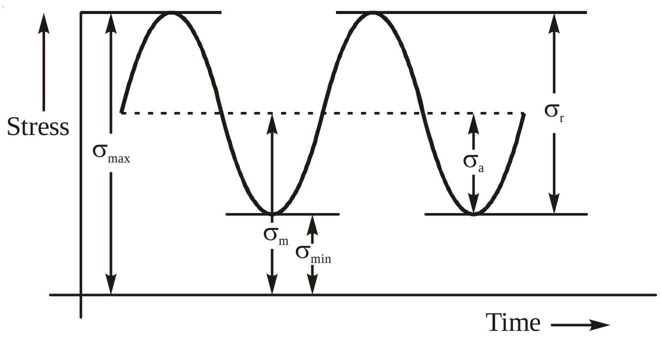

Figure-3.3: Sinusoidal fluctuating stress

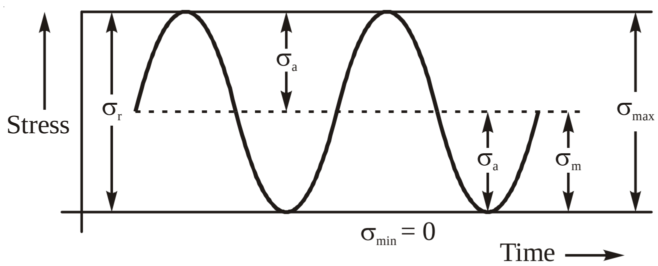

Figure-3.4: Repeated stress

Figures 3.2 to 3.4 shows the plot of *stress versus time* for

i) Sinusoidal fluctuating stress

ii) Repeated stress and

iii) Completely reversed sinusoidal stress.

Mean stress σ_m is given by,

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

Variable stress σ_a is given by,

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Figure-3.5: Completely reversed sinusoidal stress

From figure-3.4 we see that $\sigma_{\min} = 0\left(\sigma_a = \sigma_m\right)\sigma_m = \sigma_r$ for *repeated stress* and $\sigma_m = 0$ for *completely reversed sinusoidal stress (refer figure-3.5).*

Fatigue strength under fluctuating stresses

To learn the *fatigue resistance* of parts when subjected to *fluctuating stresses,* the following *methods* are in use

Modified Goodman Diagram has the *mean stress* σ_m plotted along the *abscissa* and all other *stresses* along the *ordinate.* The *endurance limit* is plotted on the *ordinate.* The *mean stress line* is 45^0 line from the origin to point *A* representing the *tensile strength* of part.

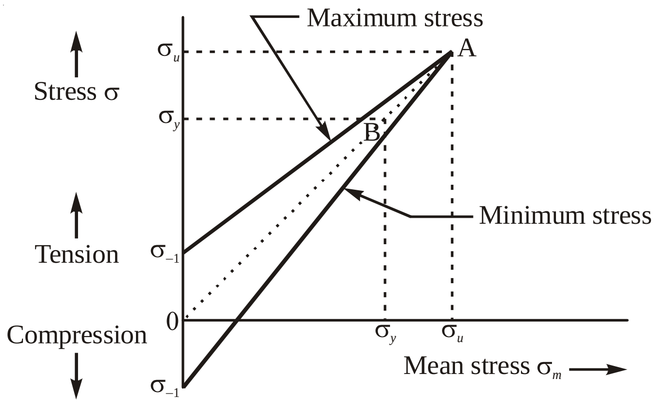

Figure-3.6: Modified Goodman Diagram

In figure-3.7, *mean stress* is plotted on the *abscissa* with *tension to the right* and *compression to the left of orign 0. Stress amplitude* σ_a is plotted on the *ordinate* the *endurance limit* or *fatigue strength* whichever is applicable to the particular problem is the *limiting value of stress amplitude* σ_a and hence it is plotted on the ordinate. A *straight line* from σ_{-1}

to σ_u on the *abscissa* is the *modified Goodman criterion of failure.* It is interesting to note that when *mean stress is tension,* most of the *failure points* fall above this line. On the compression side however the *failure points* show that the *magnitude of mean stress* has no effect.

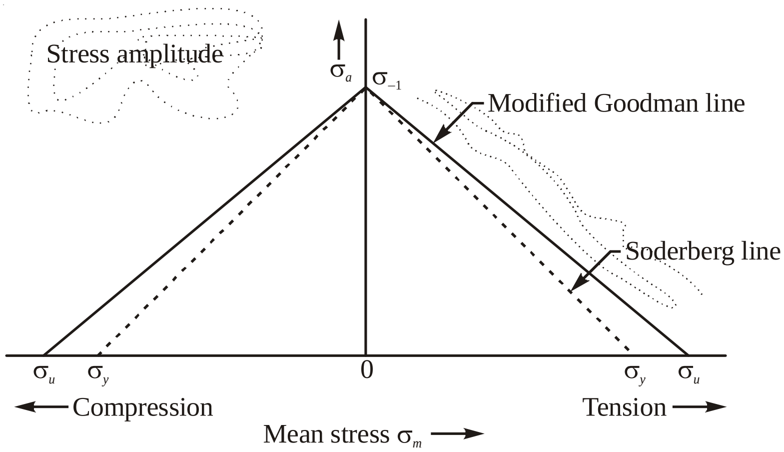

Figure-3.7

The *Soderberg line* drawn from σ_{-1} and σ_y has also been proposed as a *criterion for design* because *yielding* is used to *define failure.*

Goodman and Soderberg's relationship

From the above analysis we see that *Goodman relationship* for *fatigue loads* is based on *ultimate stress* whereas *Soderberg's equation* are based on *yield stress.*

Soderberg's relations

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad \textit{for axial loads} \qquad \text{... E(5.40)}$$

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + \frac{\sigma_{bm}}{\sigma_{yd}} = 1 \qquad \textit{for bending loads} \qquad \text{... E(5.52)}$$

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1 \qquad \textit{for torsional loads} \qquad \text{... E(5.64)}$$

Goodman's relations

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{ud}} = 1 \qquad \textit{for axial loads} \qquad \text{... E(5.39)}$$

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + \frac{\sigma_{bm}}{\sigma_{ud}} = 1 \qquad \textit{for bending loads} \qquad \text{... E(5.51)}$$

$$\frac{\tau_{ba}}{\tau_{-1d}} + \frac{\tau_m}{\tau_{ud}} = 1 \qquad \textit{for torsional loads} \qquad \text{... E(5.63)}$$

Where, $\sigma_a = Stress\ amplitude = \frac{F_a}{A}$ ... E(5.34a)

$F_a = Load\ amplitude = \frac{F_{\max} - F_{\min}}{2}$ ... E(5.32)

$F_{\max} = Maximum\ load\ on\ member$

$F_{\min} = Minimum\ load\ on\ member$

$A = Area\ of\ cross\ section\ of\ member$

$\sigma_m = Mean\ stress = \frac{F_m}{A}$ ... E(5.34b)

Where, $F_m = Mean\ load = \frac{F_{\max} + F_{\min}}{2}$ ... E(5.33)

$\sigma_{-1d} = Design\ endurance\ stress = \frac{\sigma_{-1}}{n_{-1d}}$ ... E(5.19)

Where, $\sigma_{-1} = Endurance\ stress = 50\%\ \text{of}\ \sigma_u$

$n_{-1d} = k_{sz}\ k_{sr}\ k_l\ n_{-1a}$ ... E(5.13)

Where, $k_{sz} = Size\ correction\ factor = 85\% = 0.85$

$k_{sr} = Surface\ correction\ factor = \frac{1}{e_{sr}}$

Note: e_{sr} = ***Surface coefficient. It depends on the ultimate strength of material and the surface finish.***

$k_l = Load\ factor$

$$= 0.6 \ \textit{for torsional loads}$$
$$= 0.7 \ \textit{for axial loads}$$
$$= 1.0 \ \textit{for bending loads}$$
$$n_{-1a} = \textit{Actual factor of safety}$$

$$\sigma_{yd} = \textit{Design yield stress} = \frac{\sigma_y}{n_{yd}} \quad \text{... E(5.16)}$$

Where, $n_{yd} = k_{sz} \ k_{\sigma a} \ n_{ya}$ ... E(5.12)

$$k_{\sigma a} = \textit{Stress concentration factor}$$
$$n_{ya} = \textit{Actual factor of safety}$$

$$\sigma_{ud} = \textit{Ultimate design stress} = \frac{\sigma_u}{\textit{factor of safety}}$$

$$\tau_a = \textit{Shear stress amplitude} = \frac{M_{ta}}{z_t}$$

Where, $M_{ta} = \dfrac{M_{t\max} - M_{t\min}}{2}$ ... E(5.55)

$$Z_t = \textit{Section modulus} = \frac{\pi d^3}{16} \ \textit{for torsion}$$

$$\tau_m = \textit{Mean shear stress} = \frac{M_{tm}}{z_t} \quad \text{... E(5.58)}$$

$$M_{tm} = \frac{M_{t\max} + M_{t\min}}{2} \quad \text{... E(5.56)}$$

$$\tau_{yd} = \frac{\tau_y}{n_{yd}} \ \textit{where} \ \tau_y = 0.5\sigma_y \quad \text{... E(5.17)}$$

$$\tau_{-1d} = \frac{\tau_{-1}}{n_{-1d}} \ \textit{where} \ \tau_{-1} = 25\% \ \textit{of} \ \sigma_u$$

To design a member for strength, the points to be considered are

Selection of material and *selection of design stress* to be used to calculate the proportions of the part which will be sufficiently strong. *Induced stress* in the part should always be lesser than or equal to *permissible* or *allowable* or *design stress.*

Designing of machine member for strength is one of the necessary steps in setting the proportions of a machine member. The usual steps in machine design consists of *determining the Kinematic arrangement, force analysis, selection of materials and proportioning the parts.* Proportioning of parts may be controlled by any one or all of the following namely *strength, rigidity, critical speeds, appearance, corrosion rate, fabrication, stability, etc.*

The strength of a machine member is influenced by many factors such as *stress concentration fatigue, shock surface finish and size of part.*

Derivation of Soderberg's Equation

In deriving *Soderberg's Equation, Endurance stress* σ_{-1} is plotted along *Y-axis* and *yield stress* σ_y along *X-axis.*

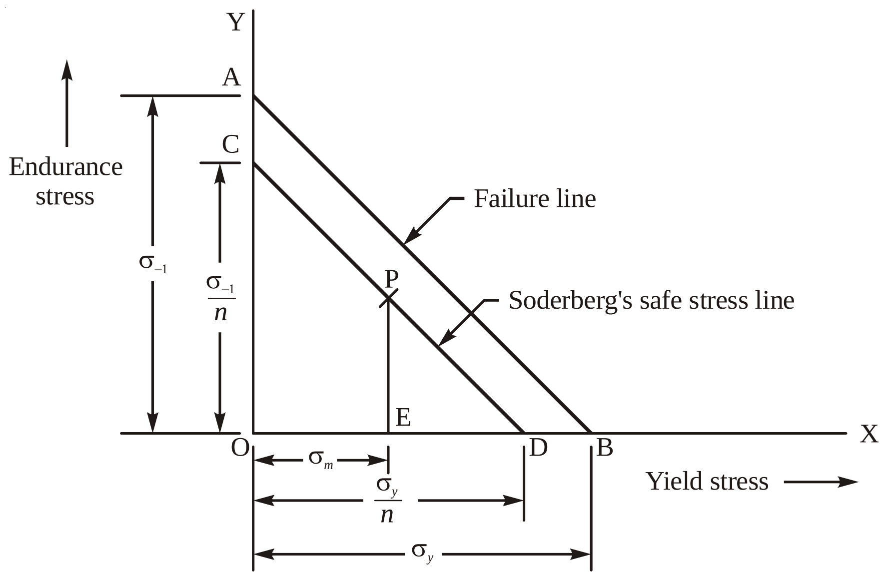

Figure-3.8

OA and *OB* represent the *endurance stress* and *yield stress* respectively. The line *AB* represents the *Soderberg's failure line.* Using *factor of safety 'n',* line *CD* is drawn parallel to *AB. AB* represents *safe stress line.* A point *'P'* on *CD* has *variable stress* σ_a on *Y-axis* and *mean stress* σ_m on *X-axis.*

From similar triangles *COD* and *PED,*

$$\frac{CO}{PE} = \frac{OD}{ED} \qquad \text{... (1)}$$

Where, $CO = \frac{\sigma_{-1}}{n} = \sigma_{-1d}$

$PE = \sigma_a$

$OD = \frac{\sigma_y}{n} = \sigma_{yd}$

And $ED = OD - OE$

$$= \frac{\sigma_y}{n} - \sigma_m = \sigma_{yd} - \sigma_m$$

Substituting in equation (1) we get,

$$\frac{\sigma_{-1d}}{\sigma_a} = \frac{\sigma_{yd}}{\sigma_{yd} - \sigma_m}$$

i.e., $$\frac{\sigma_a}{\sigma_{-1d}} = \frac{\sigma_{yd} - \sigma_m}{\sigma_{yd}}$$

i.e., $$\frac{\sigma_a}{\sigma_{-1d}} = \frac{\sigma_{yd}}{\sigma_{yd}} - \frac{\sigma_m}{\sigma_{yd}}$$

$$\frac{\sigma_a}{\sigma_{-1d}} = 1 - \frac{\sigma_m}{\sigma_{yd}}$$

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$$

This is the *Soderberg's equation* for *fluctuating axial load*

Note:* *If K_σ is the stress concentration factor, then the variable stress σ_a of E(5.40) is to be multiplied by K_σ

For fluctuation bending loads, *Soderberg's equation is*

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad \text{... E(5.52)}$$

Where, $\sigma_{ba} = Bending\ stress\ amplitude$

$$\sigma_{ba} = \frac{M_{ba}}{Z_b} \qquad \text{... E(5.45)}$$

Where, $M_{ba} = Bending\ moment\ amplitude$

$$M_{ba} = \frac{M_{b\max} - M_{b\min}}{2} \qquad \text{... E(5.43)}$$

$M_{b\max} = Maximum\ bending\ moment$

$M_{b\min} = Minimum\ bending\ moment$

$$Z_b = \frac{I}{c} = Section\ modulus = \frac{\pi d^3}{32}$$

$d = Shaft\ diameter$

$\sigma_{bm} = Mean\ bending\ stress$

$$\sigma_{bm} = \frac{M_{bm}}{Z_b} \qquad \text{... E(5.46)}$$

$M_{bm} = Mean\ bending\ moment$

$$= \frac{M_{b\max} + M_{b\min}}{2} \qquad \text{... E(5.44)}$$

For fluctuating torsional loads, Soderberg's equation is represented by,

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1$$

Where, $\tau_a = Shear\ stress\ amplitude$

$$= \frac{M_{ta}}{Z_t} \qquad \text{... E(5.57)}$$

$M_{ta} = Torque\ amplitude$

$$= \frac{M_{t\max} - M_{t\min}}{2} \qquad \text{... E(5.55)}$$

Where, $M_{t\max} = Maximum\ torque$

$M_{t\min} = Minimum\ torque$

$$Z_t = \frac{J}{c} = Section\ modulus\ in\ torsion = \frac{\pi d^3}{16}$$

$$\tau_m = Mean\ shear\ stress = \frac{M_{tm}}{Z_t} \qquad ...\ E(5.58)$$

Where, $$M_{tm} = Mean\ torque = \frac{M_{t\max} - M_{t\min}}{2} \qquad ...\ E(5.56)$$

$$\tau_{-1d} = \frac{\tau_{-1d}}{n_{-1d}} = \frac{25\%\sigma_u}{n_{-1d}}$$

$$\tau_{yd} = \frac{\tau_y}{n_{yd}} = \frac{0.6\sigma_y}{n_{yd}}$$

Example 3.1: A steel having $\sigma_u = 793.4MPa,\ \sigma_y = 552.1\%MPa$ and $\sigma_{-1} = 448.2MPa$ is subjected to an axial load fluctuating between 1kN and 3kN. Determine the diameter of the rod required based on a factor of safety 2.

Solution:

The member is subjected to fluctuating axial loads. Therefore, the diameter of the rod is calculated using Soderberg's equation for axial load given by,

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad ...\ E(5.40)$$

To find σ_a, the stress amplitude

$$\sigma_a = Stress\ amplitude = \frac{F_a}{A} \qquad ...\ E(5.34a)$$

$$F_a = Load\ amplitude = \frac{F_{\max} - F_{\min}}{2} \qquad ...\ E(5.32)$$

Where, $$F_{\max} = Maximum\ load\ on\ member = 3kN = 3\times10^3 N$$

$$F_{\min} = Minimum\ load\ on\ member = 1kN = 1\times10^3 N$$

Therefore, $$F_a = \frac{F_{\max} - F_{\min}}{2}$$

$$= \frac{3\times10^3 - 1\times10^3}{2} = 1\times10^3 N$$

$$A = Area\ of\ cross\ section\ of\ member = \frac{\pi d^2}{4}$$

$$D = Diameter\ of\ rod$$

Therefore, $\sigma_a = \frac{F_a}{A}$

$$= \frac{1\times10^3}{\frac{\pi d^2}{4}} = \frac{4000}{\pi d^2}$$

To find σ_m, the mean stress

$$\sigma_m = Mean\ stress = \frac{F_m}{A} \qquad \text{... E(5.34b)}$$

Where, $F_m = Mean\ load = \frac{F_{max} + F_{min}}{2}$ ... E(5.33)

$$= \frac{3\times10^3 + 1\times10^3}{2} = 2\times10^3 N$$

Therefore, $\sigma_m = \frac{F_m}{A}$

$$= \frac{2\times10^3}{\frac{\pi d^2}{4}} = \frac{8000}{\pi d^2}$$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = Design\ stress\ based\ on\ endurance\ limit$$

$$= \frac{\sigma_{-1}}{n_{-1d}} \qquad \text{... E(5.19)}$$

Where, $\sigma_{-1} = Endurance\ stress$

Note: In case σ_{-1} is not available in tables in data hand book, use $\sigma_{-1} = 50\%\sigma_u$

$$n_{-1d} = K_{sz} \ K_{sr} \ K_l \ n_{-1a} \qquad \text{... E(5.13)}$$

Where, $K_{sz} = Size \ factor = 0.85(assumed)$

$$K_{sr} = Surface\,correction \ factor = \frac{1}{e_{sr}} \qquad \text{... E(5.7)}$$

Where, $e_{sr} =$ Surface coefficient and is obtained from (figure 5.3MDH) based on the surface finish

Corresponding $\sigma_u = 793.4MPa,\ e_{sr} = 0.79(\text{approx})$ [assuming the rod rough finished].

Therefore, $K_{sr} = \frac{1}{e_{sr}}$

$$= \frac{1}{0.79} = 1.266$$

$K_t = load \ factor = 0.6 \ for\,Torsional\,load$

$= 0.7 \ for \ Axial \ load$

$= 1.0 \ for \ Bending \ load$

$n_{-1a} = Factor \ of \ safety = 2$

Therefore, $n_{-1d} = 0.85 \times 1.266 \times 0.7 \times 2 = 1.507$

$$\sigma_{-1d} = \frac{448.2}{1.507} = 297.41MPa$$

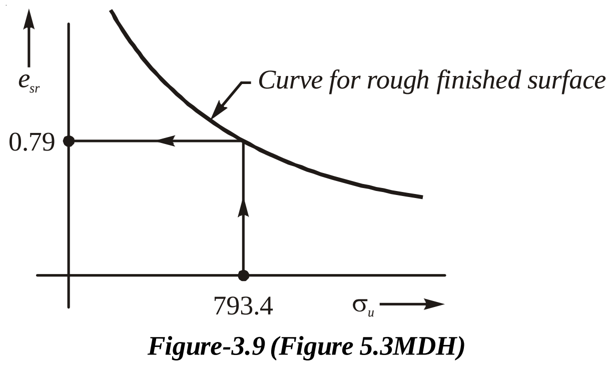

Figure-3.9 (Figure 5.3MDH)

To find σ_{yd}, the design yield stress

$$\sigma_{yd} = Design\ stress\ based\ on\ yield\ strength$$

$$= \frac{\sigma_y}{n_{yd}} \qquad \text{... E(5.16)}$$

Where, $n_{yd} = Design\ factor\ of\ safety\ based\ on\ yield\ strength$

$$= K_{sz}\ K_{\sigma a}\ n_{ya} \qquad \text{... E(5.12)}$$

$$K_{\sigma a} = Stress\ concentration\ factor$$

$$= 1,\ assuming\ no\ stress\ concentration$$

$$n_{ya} = Factor\ of\ safety = 2$$

$$n_{yd} = 0.85 \times 1 \times 2 = 1.7$$

Therefore, $\sigma_{yd} = \frac{\sigma_y}{n_{yd}}$

$$= \frac{552.1}{1.7} = 324.76 MPa$$

Substituting the values of $\sigma_a, \sigma_b, \sigma_{-1d}$ and σ_{yd} in E(5.40) we get

$$\left(\frac{\frac{4000}{\pi d^2}}{297.41}\right) + \left(\frac{\frac{8000}{\pi d^2}}{324.76}\right) = 1$$

$$\frac{4.28}{d^2} + \frac{7.84}{d^2} = 1$$

Therefore, *Diameter of rod, d = 3.48mm.*

Example 3.2: A steel rod having $\sigma_u = 793.4 MPa$, $\sigma_y = 552.1 MPa$ and $\sigma_{-1} = 448.2 MPa$ with rough finished surface is subjected to a completely reversed axial load of 2kN. Determine the diameter of the rod based on factor of safety of 3. Use Soderberg's equation.

Solution:

For fluctuating axial load, the Soderberg's equation is

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad \text{... E(5.40)}$$

Where, $\sigma_a = \dfrac{F_a}{A}$

$$= \frac{\frac{F_{\max} - F_{\min}}{2}}{\frac{\pi d^2}{4}}; \qquad A = \frac{\pi d^2}{4}$$

Note: For a completely reversed axial load of 2kN.

$$F_{\max} = 2kN = 2\times10^3 N$$

$$F_{\min} = -2kN = -2\times10^3 N$$

Therefore, $\sigma_a = \dfrac{\frac{2\times10^3 - \left(-2\times10^3\right)}{2}}{\frac{\pi d^2}{4}}$

Where, *d = diameter of the rod*

$$\sigma_a = \frac{2\times10^3\times4}{\pi d^2} = \frac{8\times10^3}{\pi d^2}$$

$$\sigma_m = \frac{F_m}{A}$$

$$= \frac{\frac{F_{\max} + F_{\min}}{2}}{\frac{\pi d^2}{4}} = \frac{\frac{2\times10^3 + \left(-2\times10^3\right)}{2}}{\frac{\pi d^2}{4}}$$

i.e, $\sigma_m = 0$

Note: For a completely reversed load, mean stress = 0.

i.e., $\sigma_m = 0 \;\; for\ Axial\ loads$

$\sigma_{bm} = 0 \;\; for\ Bending\ loads$

And $\tau_m = 0 \;\; for\ Torsional\ loads$

Therefore, the second term $\frac{\sigma_m}{\sigma_{yd}}$ of Soderberg's equation becomes zero, and the equation gets reduced to

$$\frac{\sigma_a}{\sigma_{-1d}} + 0 = 1 \quad or \quad \sigma_a = \sigma_{-1d}$$

To find σ_{-1d}

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

$$n_{-1d} = K_{sz}\ K_{sr}\ K_l\ n_{-1a}$$

$$K_{sz} = 0.85$$

$$K_{sr} = \frac{1}{e_{sr}}$$

$e_{sr} = 0.79$ (approximate) corresponding to $\sigma_u = 793.4MPa$

(Assuming the surface of rod rough finished)

$$K_l = 0.7$$

$$n_{-1a} = 3$$

$$\sigma_{-1d} = \frac{448.2}{0.85 \times \frac{1}{0.79} \times 0.7 \times 3} = 198.36MPa$$

Equating σ_a and σ_{-1d} we get,

$$\frac{8 \times 10^3}{\pi d^2} = 198.36$$

Therefore, *Diameter of rod, d = 3.58mm.*

Example 3.3: A steel rod having $\sigma_u = 620.8MPa$, $\sigma_y = 400.1MPa$ and $\sigma_{-1} = 345.2MPa$ is subjected to bending moment fluctuating between 200Nm and 400Nm. Determine the diameter of the rod required based on a factor of safety of 2.5. (VTU-Dec08 / Jan09)

Solution:

For *fluctuating bending loads, the Soderberg's equation is given by,*

$$\frac{\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \qquad \text{... E(5.52)}$$

To find σ_{ba}, the bending stress amplitude

$$\sigma_{ba} = \textit{Bending stress amplitude} = \frac{M_{ba}}{Z_b} \qquad \text{... E(5.45)}$$

Where, $M_{ba} = \textit{Bending moment amplitude} = \dfrac{M_{b\max} - M_{b\min}}{2}$ ... E(5.43)

$$M_{b\max} = 400\text{Nm} = 400 \times 10^3 \text{Nmm}$$

$$M_{b\min} = 200\text{Nm} = 200 \times 10^3 \text{Nmm}$$

Therefore, $M_{ba} = \dfrac{400 \times 10^3 - 200 \times 10^3}{2} = 100 \times 10^3 \text{Nmm}$

$$Z_b = \frac{I}{c} = \frac{\pi d^3}{32}$$

Where, *d=diameter of rod required.*

Therefore, $\sigma_{ba} = \dfrac{M_{ba}}{Z_b}$

$$= \frac{100 \times 10^3}{\frac{\pi d^3}{32}} = \frac{32 \times 10^5}{\pi d^3}$$

To find σ_{bm}, the mean bending stress

$$\sigma_{bm} = \textit{Mean bending stress} = \frac{M_{bm}}{Z_b}$$

Where, $M_{bm} = \textit{Mean bending moment}$

$$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$

$$= \frac{400 \times 10^3 + 200 \times 10^3}{2}$$

$$= 300 \times 10^3 \text{ Nmm}$$

Therefore, $\sigma_{bm} = \dfrac{300 \times 10^3}{\dfrac{\pi d^3}{32}} = \dfrac{96 \times 10^5}{\pi d^3}$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

Where, $n_{-1d} = K_{sz} \ K_{sr} \ K_l \ n_{-1a}$

$$K_{sz} = 0.85 (assumed)$$

$$K_{sr} = \frac{1}{e_{sr}}$$

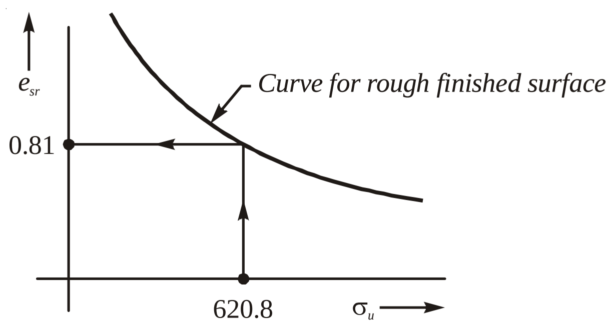

Figure-5.3MDH

Corresponding to $\sigma_u = 620.8 MPa$, for rough finished steel rod, $e_{sr} = 0.81 (approx)$

$K_l = load \ \ factor = 1$ (for bending load)

$$n_{-1a} = 2.5$$

Therefore, $n_{-1d} = 0.85 \times \dfrac{1}{0.81} \times 1 \times 2.5 = 2.623$

And $$\sigma_{-1d} = \frac{345.2}{2.623} = 131.58MPa$$

To find $\sigma_{yd,}$ the design yield stress

$$\sigma_{yd} = \frac{n_y}{n_{yd}} = \frac{\sigma_y}{K_{sz}\,K_{\sigma a} n_{ya}}$$

$$K_{\sigma a} = \text{Stress concentration factor} = 1$$

(Since there is no stress concentration)

$$K_{\sigma a} = \frac{400.1}{0.85 \times 1 \times 2.5} = 188.28MPa$$

Substituting the above values in E(5.52) we get,

$$\left(\frac{\frac{32 \times 10^5}{\pi d^3}}{131.58} \right) + \left(\frac{\frac{95 \times 10^5}{\pi d^3}}{188.28} \right) = 1$$

Therefore, *Diameter of the rod required, d=28.83mm.*

Example 3.4: A rough finished steel rod having $\sigma_u = 620.8MPa$, $\sigma_y = 400.1MPa$ and $\sigma_{-1} = 345.2MPa$ is subjected to a completely reversed bending moment of 400N-m. Determine the diameter of the rod required based on a factor of safety of 2.5.

Solution:

For fluctuating axial load, the Soderberg's equation is

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + \frac{\sigma_{bm}}{\sigma_{yd}} = 1 \qquad \text{... E(5.52)}$$

Where, $$\sigma_{ba} = \frac{M_{ba}}{Z_b}$$

$$= \frac{\frac{M_{b\max} - M_{b\min}}{2}}{\frac{\pi d^2}{4}}$$

$$M_{b\max} = 400\,\text{N-m} = 400\times10^3\,\text{N-mm}$$

$$M_{b\min} = -400\,\text{N-m} = -400\times10^3\,\text{N-mm}$$

$$\sigma_{ba} = \frac{M_{ba}}{Z_b}$$

Where, $Z_b = Section\ modulus = \frac{I}{c} = \frac{\pi d^3}{32}$

$$\sigma_{ba} = \frac{\frac{M_{b\max} - M_{b\min}}{2}}{\frac{\pi d^3}{32}}$$

Therefore, $\sigma_{ba} = \frac{400\times10^3\times32}{\pi d^3} = \frac{128\times10^5}{\pi d^3}$

$$\sigma_{bm} = \frac{M_{bm}}{Z_b}$$

$$= \frac{\frac{M_{b\max} + M_{b\min}}{2}}{\frac{\pi d^3}{32}}$$

$$\sigma_{bm} = \frac{\frac{400\times10^3 + \left(-400\times10^3\right)}{2}}{\frac{\pi d^3}{32}}$$

$$\sigma_{bm} = 0$$

Therefore, Soderberg's equation E(5.52) gets reduced to

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + 0 = 1 \quad i.e, \quad \sigma_{ba} = \sigma_{-1d}$$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

$$= \frac{\sigma_{-1}}{K_{sz}\ K_{sr}\ K_l\ n_{-1a}}$$

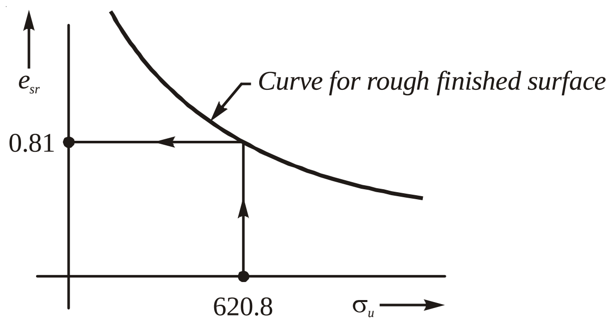

Figure-5.3 MDH

$K_{sz} = 0.85,\ K_{sr} = \frac{1}{e_{sr}}$ and e_{sr} is obtained from figure-5.3MDH. Corresponding to $\sigma_u = 620.8 MPa,\ e_{sr} = 0.81,\ K_l = 1.0,\ n_{-1a} = 2.5$

Therefore, $$\sigma_{-1d} = \frac{345.2}{0.85 \times \frac{1}{0.81} \times 1 \times 2.5} = 131.58 MPa$$

Equating σ_{ba} and σ_{-1d} we get,

$$\frac{128 \times 10^5}{\pi d^2} = 131.58$$

Therefore, *Diameter of rod required, d = 31.4mm.*

Example 3.5: A cantilever beam shown in figure-3.10 is subjected to load varying from P to 3P. Determine the value of P. The material of the beam has $\sigma_u = 620.8MPa$, $\sigma_y = 400.1MPa$ and $\sigma_{-1} = 345.2MPa$. The stress concentration factor may be taken as 1.4. Analyze the member at the change of cross section A-A. Use factor of safety = 3.

(VTU-Dec 06 / Jan 07)

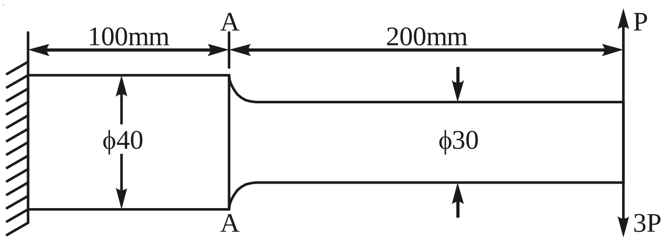

Figure-3.10

Solution:

The member is subjected to fluctuating bending loads.

At the change of cross section A-A, we have,

Maximum bending moment $M_{b\max} = 3P \times 200 = 600P$ N-mm

Minimum bending moment $M_{b\min} = -P \times 200 = -200P$ N-mm

Soderberg's equation for bending load is,

$$\frac{\sigma_{ba}}{\sigma_{-1d}} + \frac{\sigma_{bm}}{\sigma_{yd}} = 1 \quad \text{... E(5.52)}$$

Where, $$\sigma_{ba} = \frac{M_{ba}}{Z_b}$$

$$= \frac{\frac{M_{b\max} - M_{b\min}}{2}}{\frac{\pi d^2}{4}}$$

Here, $d = 30mm\left(minimum\ diameter\right)$

$$\sigma_{ba} = \frac{\frac{600P - \left(-200P\right)}{2}}{\frac{\pi d^3}{32}}$$

$$= \frac{400 \times 32P}{\pi\left(30\right)^3}$$

$$= \frac{12800P}{\pi\left(30\right)^3} = 0.1509P$$

$$\sigma_{bm} = \frac{M_{bm}}{Z_b}$$

$$= \frac{\frac{M_{b\max} + M_{b\min}}{2}}{\frac{\pi d^3}{32}}$$

$$= \frac{\frac{600P + \left(-200P\right)}{2}}{\frac{\pi d^3}{32}} = \frac{6400}{\pi d^3}$$

$$= \frac{6400P}{\pi\left(30\right)^3} = 0.07545P$$

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

$$= \frac{\sigma_{-1}}{K_{sz}\ K_{sr}\ K_l\ n_{-1a}}$$

$$K_{sz} = 0.85,\ K_{sr} = \frac{1}{e_{sr}} = \frac{1}{0.81},\ K_l = 1,\ n_{-1a} = 3$$

$e_{sr} = 0.81$ corresponding to $\sigma_u = 620.8MPa$ (Figure-5.3 MDH)

Therefore, $$\sigma_{-1d} = \frac{345.2}{0.85 \times \frac{1}{0.81} \times 1 \times 3} = 109.65MPa$$

$$\sigma_{yd} = \frac{\sigma_y}{n_{yd}}$$

$$= \frac{\sigma_y}{K_{sz}\ K_{\sigma a}\ \ n_{ya}}$$

$$= \frac{400.1}{0.85 \times 1.4 \times 3} = 112.07MPa$$

Substituting the values in E(5.22) we get,

$$\frac{0.1509P}{109.65} + \frac{0.07545P}{112.07} = 1$$

$$P = 487.94N$$

Example 3.6: A carbon steel shaft with $\sigma_u = 600MPa,\ \sigma_y = 330MPa$ is subjected to torsional load varying from 200N-m and 400N-m. Determine the diameter of the shaft required based on a factor of safety of 3. Base the design on Soderberg's equation.

Solution:

The shaft is subjected to fluctuating torsional load.

For *fluctuating torsional loads, Soderberg's equation is,*

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1 \qquad \text{... E(5.64)}$$

To find τ_a, the shear stress amplitude

$$\tau_a = \text{Shear stress amplitude} = \frac{M_{ta}}{Z_t} \qquad \text{... E(5.57)}$$

Where, $$M_{ta} = \frac{M_{t\max} - M_{t\min}}{2} \qquad \text{... E(5.43)}$$

$$M_{t\max} = 400 \text{ N-m} = 400 \times 10^3 \text{ N-mm}$$

$$M_{t\min} = 200 \text{ N-m} = 200 \times 10^3 \text{ N-mm}$$

$$M_{ta} = \frac{400 \times 10^3 - 200 \times 10^3}{2} = 100 \times 10^3 \text{ N-mm}$$

$$Z_t = \textit{Section modulus in torsion} = \frac{J}{c}$$

$$= \frac{\pi d^3}{16}; \quad \textit{where } d = \textit{diameter of shaft}$$

Therefore, $$\tau_a = \frac{100 \times 10^3}{\left(\frac{\pi d^3}{16}\right)} = \frac{16 \times 10}{\pi d^3}$$

To find τ_m, the mean shear stress

$$\tau_m = Mean\ shear\ stress = \frac{M_{tm}}{Z_t} \qquad \text{... E(5.58)}$$

Where, $$M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$$

$$= \frac{400 \times 10^3 + 200 \times 10^3}{2} = 300 \times 10^3 \text{ N-mm}$$

$$\tau_m = \frac{300 \times 10^3}{\frac{\pi d^3}{16}} = \frac{48 \times 10^5}{\pi d^3}$$

To find τ_{-1d}, the design endurance shear stress

$$\tau_{-1d} = \frac{\tau_{-1}}{n_{-1d}}$$

Note: **τ_{-1d} *is not available in data hand book tables and is generally assumed 25% σ_u***

Therefore, $$\tau_{-1d} = 0.25\sigma_u$$

$$= 0.25 \times 600 = 150 MPa$$

$$n_{-1d} = K_{sz}\ \ K_{sr}\ \ K_l\ \ n_{-1a}$$

Where, $$K_{sz} = 0.85(assumed)$$

$$K_{sr} = \frac{1}{e_{sr}} \qquad \text{... E(5.7)}$$

And e_{sr} is obtained from equation,

$$e'_{sr} = 0.425 + 0.575 e_{sr}\ (for\ torsion) \qquad \text{... E(5.8)}$$

e'_{sr} is obtained from figure-5.3MDH; $e'_{sr} = 0.91$

Substituting in E(5.8)

$$0.91 = 0.425 + 0.575\ e_{sr}$$

Therefore, $e_{sr} = 0.8434$

$$n_{-1d} = 0.85 \times \frac{1}{0.8434} \times 0.6 \times 3 = 1.841$$

Therefore, $\tau_{-1d} = \frac{150}{1.814} = 82.69 MPa$

To find τ_{yd}, the design yield shear stress

$$\tau_{yd} = Design\ yield\ shear\ stress = \frac{\tau_y}{n_{yd}} \qquad \text{... E(5.17)}$$

Where, $\tau_y = 0.5\ to\ 0.6\sigma_y; \qquad say\ \tau_y = 0.5\sigma_y$

Therefore, $\tau_y = 0.5 \times 330 = 165 MPa$

$$n_{yd} = K_{sz}\ K_{\sigma a}\ n_{ya}$$

$$= 0.85 \times 1 \times 3 = 2.55$$

Therefore, $\tau_{yd} = \frac{165}{2.55} = 64.71 MPa$

Substituting these values in E(5.64) we get,

$$\left(\frac{\frac{16 \times 10^5}{\pi d^3}}{82.69} \right) + \left(\frac{\frac{48 \times 10^5}{\pi d^3}}{64.71} \right) = 1$$

$$d = 30.99 mm$$

Example 3.7:A carbon steel shaft with $\sigma_u = 600 MPa$, $\sigma_y = 330 MPa$ is subjected to bending moment varying from 100N-m and 200N-m and torsional moment varying from 200N-m and 400N-m. The maximum bending moment occurs at the same time as that of maximum torsional moment. Determine the diameter of the shaft required based on a factor of safety of 3.

Solution:

The diameter of the shaft subjected to combined fluctuating torsion and bending loads is given by,

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(M'_{bm}\right)^2 + \left(M'_{tm}\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14.21)}$$

Where, *D = Shaft diameter.*

$$M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{-1d}} M_{ba} \qquad \text{... E(14.22a)}$$

$$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$

$$= \frac{200 \times 10^3 + 100 \times 10^3}{2} = 150 \times 10^3 \text{ N-mm}$$

$$\sigma_{sd} = \frac{\sigma_s}{factor\ of\ safety}$$

$$= \frac{330}{3} = 110 MPa$$

Therefore, $M'_{bm} = 150 \times 10^3 + \frac{110}{100} \times 50 \times 10^3$

$$= 205 \times 10^3 \text{ N-mm}$$

$$M'_{tm} = M_{tm} + \frac{\tau_{sd}}{\tau_{-1d}} M_{ta} \qquad \text{... E(14.22b)}$$

Where, $M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$

$$= \frac{400 \times 10^3 + 200 \times 10^3}{2} = 300 \times 10^3 \text{ N-mm}$$

$$M_{ta} = \frac{M_{t\max} - M_{t\min}}{2}$$

$$= \frac{400 \times 10^3 - 200 \times 10^3}{2} = 100 \times 10^3 \text{ N-mm}$$

$$\tau_{sd} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{0.6 \times 330}{3} = 66MPa \quad \left(\text{since } \tau_y = 0.6\sigma_y\right)$$

$$\tau_{-1d} = \frac{\tau_{-1}}{factor\ of\ safety}$$

$$= \frac{0.25\sigma_u}{factor\ of\ safety}$$

$$\frac{0.25 \times 600}{3} = 50MPa$$

Therefore, $$M'_{tm} = 300 \times 10^3 + \frac{66}{50} \times 100 \times 10^3$$

$$= 432 \times 10^3 mm$$

$$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{0.6\sigma_y}{factor\ of\ safety}$$

$$= \frac{0.6 \times 330}{3} = 66MPa$$

Substituting these values in E(14.21) we get,

$$D = \left[\frac{16}{\pi \times 66} \left\{ \left(205 \times 10^3\right)^2 + \left(432 \times 10^3\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Therefore, *the diameter of shaft D = 33.29mm.*

Example 3.8: A steel shaft with $\sigma_u = 600MPa,\ \sigma_y = 330MPa$ is subjected to bending moment fluctuating from 200N-m and 100N-m when the torsional moment varies from 400N-m in clockwise direction to 200N-m in the anti-clockwise direction. Determine the diameter of the shaft required based on factor of safety of 2.

Solution:

The diameter of the shaft subjected to combined fluctuating torsion and bending loads is given by,

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(M'_{bm} \right)^2 + \left(M'_{tm} \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \quad \text{... E(14.21)}$$

Where, *D = Shaft diameter.*

$$M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{-1d}} M_{ba} \quad \text{... E(14.22a)}$$

$$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$

$$M_{b\max} = 200\text{N-mm} = 200 \times 10^3 \text{N-mm}$$

$$M_{b\min} = -100\text{N-mm} = -100 \times 10^3 \text{N-mm}$$

$$M_{bm} = \frac{200 \times 10^3 + \left(-100 \times 10^3 \right)}{2}$$

$$= 50 \times 10^3 \text{N-mm}$$

$$M_{ba} = \frac{M_{b\max} - M_{b\min}}{2}$$

$$= \frac{200 \times 10^3 - \left(-100 \times 10^3 \right)}{2} = 150 \times 10^3 \text{N-mm}$$

$$\sigma_{sd} = \frac{\sigma_y}{factor\ of\ safety}$$

$$= \frac{330}{2} = 165 MPa$$

$$\sigma_{-1d} = \frac{\sigma_{-1}}{factor\ of\ safety}$$

$$= \frac{0.5 \sigma_u}{factor\ of\ safety}$$

$$= \frac{0.5 \times 600}{3} = 150 MPa$$

Therefore, $M'_{bm} = 50\times10^3 + \frac{165}{150}\times150\times10^3$

$= 215\times10^3$ N-mm

Similarly, $M'_{tm} = M_{tm} + \frac{\tau_{sd}}{\tau_{-1d}} M_{ta}$ ... E(14.22b)

Where, $M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$

Assuming clockwise moments as positive and anticlockwise moments as negative

$M_{t\max} = +400$ N-m $= 400\times10^3$ N-mm

$M_{t\min} = -200$ N-m $= -200\times10^3$ N-mm

$$\tau_{sd} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{0.6\sigma_y}{factor\ of\ safety}$$

$$= \frac{0.6\times600}{2} = 99 MPa$$

$$\tau_{-1d} = \frac{\tau_{-1}}{factor\ of\ safety}$$

$$= \frac{0.25\sigma_u}{factor\ of\ safety}$$

$$\frac{0.25\times600}{2} = 75 MPa$$

$$M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$$

$$= \frac{400\times10^3 + \left(-200\times10^3\right)}{2} = 100\times10^3 \text{ N-mm}$$

$$M_{ta} = \frac{M_{t\max} - M_{t\min}}{2}$$

$$= \frac{400\times10^3-\left(-200\times10^3\right)}{2} = 300\times10^3\,\text{N-mm}$$

Therefore, $$M'_{tm} = 100\times10^3 + \frac{99}{75}\times300\times10^3$$

$$= 496\times10^3\,\text{N-mm}$$

$$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{0.6\sigma_y}{factor\ of\ safety}$$

$$= \frac{0.6\times330}{2} = 99MPa$$

Substituting these values in E(14.21) we get,

$$D = \left[\frac{16}{\pi\times99}\left\{\left(215\times10^3\right)^2+\left(496\times10^3\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

Therefore, *the diameter of shaft D = 30.29mm.*

Example 3.9: A steel shaft having ultimate stress = 600MPa, yield stress = 400MPa is subjected to bending moment fluctuating from 100N-m to 200N-m when the axial tensile load varies from 10kN and 20kN. The maximum bending moment occurs at the instant when the maximum load occurs. Determine the diameter of shaft required based on a factor of safety of 2.

Solution:

The diameter of the shaft subjected to fluctuating axial, bending and torsonial loads is given by,

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(M'_{bm}+\frac{\alpha F'_m D}{8}\right)^2+M'^2_{tm}\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14.24)}$$

Where, D = *Shaft diameter*

Now, $$M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{-1d}} M_{ba}$$

Where, $$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$

$$= \frac{200\times10^3 + 100\times10^3}{2} = 150\times10^3 \text{ N-mm}$$

$$M_{ba} = \frac{M_{b\max} - M_{b\min}}{2}$$

$$= \frac{200\times10^3 - 100\times10^3}{2} = 50\times10^3 \text{ N-mm}$$

$$\sigma_{sd} = \frac{\sigma_y}{factor\ of\ safety}$$

$$= \frac{400}{2} = 200 MPa$$

$$\sigma_{-1d} = \frac{\sigma_{-1}}{factor\ of\ safety}$$

$$= \frac{0.5\times600}{2} = 150 MPa \qquad (\text{Assuming } \sigma_{-1} = 0.5\sigma_u)$$

Therefore, $$M'_{bm} = 150\times10^3 + \frac{200}{150}\times50\times10^3$$

$$= 216.67\times10^3 \text{ N-mm}$$

$M'_{tm} = 0$ since no torsional load acts on the shaft.

$\alpha = Column\ factor = 1\ for\ axial\ tensile\ load$

Note: ***For axial compressive loads on the shaft, α is found using equations (14.64) or (14.65) depending on slenderness ratio $\frac{l}{K}$.***

Now, $$F'_m = F_m + \frac{\sigma_{sd}}{\sigma_{-1d}} F_a$$

Where, $$F_m = \frac{F_{\max} + F_{\min}}{2}$$

$$= \frac{20\times10^3 + 10\times10^3}{2} = 15\times10^3 \text{ N-mm}$$

$$F_a = \frac{F_{\max} - F_{\min}}{2}$$

$$= \frac{20\times10^3 - 10\times10^3}{2} = 5\times10^3 N$$

$$\sigma_{sd} = \frac{\sigma_y}{factor\ \ of\ \ safety}$$

$$= \frac{400}{2} = 200MPa$$

Therefore, $$F'_m = 15\times10^3 + \frac{200}{150}\times50\times10^3$$

$$= 21.67\times10^3 \text{ N-mm}$$

$$\tau_{ed} = \frac{\tau_y}{factor\ \ of\ \ safety}$$

$$= \frac{0.5\sigma_y}{factor\ \ of\ \ safety}$$

$$= \frac{0.5\times400}{2} = 100MPa$$

Substituting in E(12.24) we get

$$D = \left[\frac{16}{\pi\times100}\left\{216.67\times10^3 + \frac{1\times21.67\times10^3 D}{8}\right\}\right]^{\frac{1}{3}}$$

Cubing both the sides we get,

$$D^3 = \frac{16}{\pi \times 100}\left\{\frac{216.67\times10^3 + 2.709\times10^3 D}{8}\right\}$$

By trial and error, the shaft diameter D = 28mm.

Example 3.10: Determine the safe torque that can be transmitted by a solid circular shaft 60mm diameter with a semi circular groove of 5mm radius limiting the maximum shear stress induced in the shaft to $100\, N/mm^2$ ***. Also determine the safe power in kW that can be transmitted at 750rpm.***

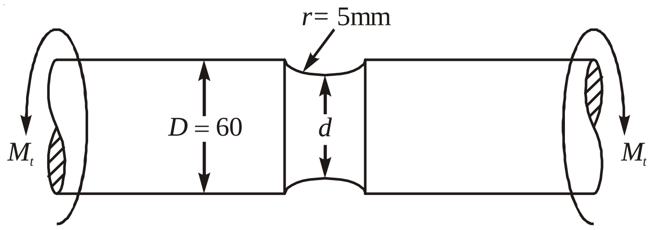

Figure-3.11

Solution:

Refer figure-4.22 MDH

For the given shaft, $D = 60mm$

$d = 60 - 2\times5 = 50mm$

$r = \ fillet\ radius = 5mm$

Therefore, $\frac{r}{d} = \frac{5}{50} - 0.1$

And $\frac{D}{d} = \frac{60}{50} = 1.2$

Corresponding to $\frac{D}{d} = 1.2,\ \frac{r}{d} = 0.1,$ the *stress concentration factor in torsion,* $K_\tau = 1.45$ [from figure-4.18A]

We have, *Stress concentration factor for torsion*

$$K_\tau = \frac{\tau_{\max}}{\tau_{nominal}}$$

Where, $\tau_{\max} = 100\, N/mm^2$

$K_\tau = 1.45$

Therefore, $\tau_{nominal} = \frac{100}{1.45} = 68.97\,N/mm^2$

But, $\tau_{nominal} = \frac{M_t}{Z_t)_{\min}}$

Since, $Z_t)_{\min} = \frac{\pi d^3}{16}$

$$\tau_{nominal} = \frac{16M_t}{\frac{\pi d^3}{16}} = \frac{16M_t}{\pi d^3}$$

$$M_t = \frac{\tau_{nominal} \times \pi d^3}{16}$$

$$= \frac{68.97 \times \pi (50)^3}{16}$$

$$= 1692778.5 \text{ N-mm} = 1692.78 \text{ N-m}$$

To determine the power transmitted (N)

Torque, $M_t = \frac{9550N}{n}$ N-m

Power, $N = \frac{M_t n}{9550} kW$

$$= \frac{1692.78 \times 750}{8550}$$

Therefore, $N = 132.94kW$

Example 3.11: A section of a shaft of diameter 'd' is joined to another section of a shaft of diameter 1.5d with a fillet which produces an actual stress concentration factor of 1.2 for the shaft is torsion. The material has a yield point in torsion of 550MPa and endurance limit of 288MPa in reversed torsion. Using size factor 0.85, surface finish factor = 0.9 and load factor of 0.6, determine the size of shaft required for a torque which varies from zero to 2400N-m on the shaft at the critical section. Use design factor of 2. (VTU-March / April 2000)

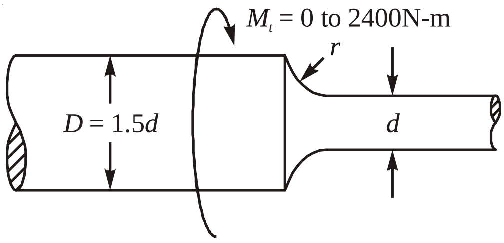

Figure-3.12

Solution:

For fluctuating torsional loads, the Soderberg's equation is,

$$\frac{\tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1 \qquad \text{... E(5.64)}$$

Where,
$$\tau_a = \frac{M_{ta}}{Z_t} = \frac{M_{t\max} - M_{t\min}}{2Z_t} \qquad \text{... E(5.57)}$$

$$M_{ta} = \frac{2400 \times 10^3 - 0}{2Z_t} = 1200 \times 10^3 \text{ N-mm}$$

$$Z_t = \frac{\pi d^3}{16}$$

Therefore,
$$\tau_a = \frac{1200 \times 10^3}{\frac{\pi d^3}{16}}$$

$$= \frac{16 \times 1200 \times 10^3}{\pi d^3}$$

Now,
$$\tau_m = \frac{M_{tm}}{Z_t} \qquad \text{... E(5.58)}$$

Where,
$$M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$$

$$= \frac{2400 \times 10^3 + 0}{2} = 1200 \times 10^3 \text{ N-mm}$$

Therefore, $\tau_m = \dfrac{1200\times10^3}{\dfrac{\pi d^3}{16}}$

$$= \frac{19200\times10^3}{\pi d^3}$$

We have, $\tau_{-1d} = \dfrac{\tau_{-1}}{n_{-1d}}$

Where, $\tau_{-1} = 288\, N/mm^2$

$$n_{-1d} = K_{sz}\ \ K_{sr}\ \ K_l\ \ n_{-1a}$$

$$= 0.85\times0.9\times0.6\times2$$

$$= 0.918$$

Therefore, $\tau_{-1d} = \dfrac{288}{0.918} = 311.73\, N/mm^2$

Finally, $\tau_{yd} = \dfrac{\tau_y}{n_{yd}}$

Where, $\tau_y = 550\, N/mm^2$

$$n_{yd} = K_{sz}\ \ K_{\sigma a}\ \ n_{ya}$$

$$= 0.85\times1.2\times2 = 2.04$$

Therefore, $\tau_{yd} = \dfrac{5501}{2.04} = 269.61\, N/mm^2$

Substituting these values in Soderberg's equation we get,

$$\left(\frac{\dfrac{19200\times10^3}{\pi d^3}}{313.73}\right) + \left(\frac{\dfrac{19200\times10^3}{\pi d^3}}{269.61}\right) = 1$$

Therefore, *Adopt d = 35mm and* $D = 1.5d = 52.5mm$

Example 3.12: A cantilever beam of span 2000mm having rectangular cross section with depth 80mm is subjected to a transverse load at its free end fluctuating between 800N downwards to 2000N upwards. The material selected for the beam has a value of

yield stress $300\,N/mm^2$ ***and endurance stress*** $250\,N/mm^2$. ***Taking factor of safety 2.25, determine the width of rectangular cross section of beam.*** ***(VTU-Exam Question)***

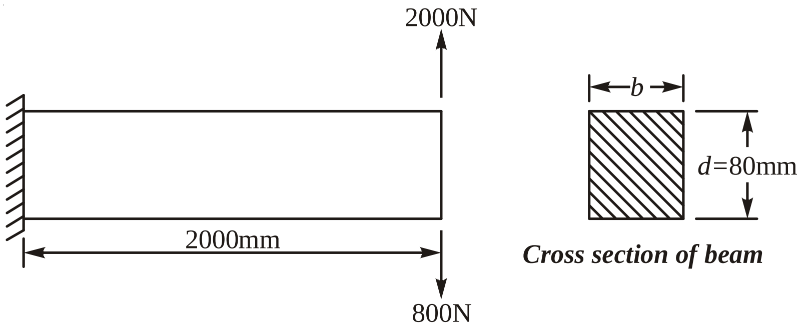

Figure-3.13

Solution:

The cantilever beam is subjected to fluctuating bending loads. Soderberg's equation for bending loads is given by,

$$\frac{\sigma_{ba}}{\sigma_{-1d}}+\frac{\sigma_{bm}}{\sigma_{yd}}=1 \qquad \text{... E(5.52)}$$

To find σ_{ba}***, the bending stress amplitude***

$$\sigma_{ba}=\frac{M_{ba}}{Z_b} \qquad \text{... E(5.45)}$$

Where, $$M_{ba}=\frac{M_{b\max}-M_{b\min}}{2} \qquad \text{... E(5.43)}$$

$$M_{b\max}=200\times 200\text{ N-mm}=4\times 10^6\text{ N-mm}$$

$$M_{b\min}=-800\times 200\text{ N-mm}=-16\times 10^5\text{ N-mm}$$

Therefore, $$M_{ba}=\frac{4\times 10^6-\left(-16\times 10^5\right)}{2}$$

$$=28\times 10^5\text{ N-mm}$$

$$Z_b=\frac{I}{c}=\frac{bd^2}{6}$$

$$=\frac{b\times 80^2}{6}=1066.68b$$

$$\sigma_{ba} = \frac{28\times10^5}{1066.67b} = \frac{2625}{b}$$

Also, $$\sigma_{bm} = \frac{M_{bm}}{Z_b}$$ ... E(5.46)

Where, $$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$ ... E(5.44)

$$= \frac{4\times10^6 + \left(-16\times10^5\right)}{2} = 12\times10^5 \text{ N-mm}$$

$$\sigma_{bm} = \frac{12\times10^5}{1066.67b} = \frac{1124.996}{b}$$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$ ... E(5.19)

$$\sigma_{-1} = 250\, N/mm^2$$

$$n_{-1d} = K_{sz}\ \ K_{sr}\ \ K_l\ \ n_{-1a}$$ ... E(5.13)

Where, $$K_{sz} = 0.85$$

$$K_{sr} = \frac{1}{e_{sr}}$$

$$K_l = 1\left(for\ bending\right)$$

$$n_{-1a} = 2.25\left(given\right)$$

To find K_{sr}, the surface correction factor

Taking, $$\sigma_u = 2\sigma_{-1}$$

$$= 2\times250\, N/mm^2 = 500\, N/mm^2 = 500MPa$$

From figure-5.3MDH, for $\sigma_u = 500MPa,\ e_{sr} = 0.84$ (for rough finished rod)

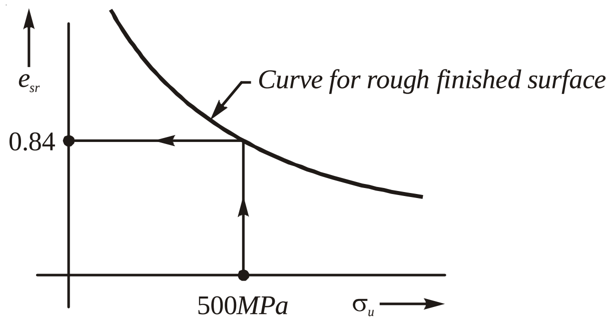

Figure-3.14

Therefore, $K_{sr} = \dfrac{1}{e_{sr}}$

$= \dfrac{1}{0.84} = 1.1905$

i.e., $n_{-1d} = 0.85 \times 1.1905 \times 1 \times 2.25$

$= 2.27683$

Therefore, $\sigma_{-1d} = \dfrac{250}{2.27683} = 109.8\, N/mm^2$

Finally, $\sigma_{yd} = \dfrac{\sigma_y}{n_{yd}}$ ... E(5.16)

Where, $\sigma_y = 300\, N/mm^2\ (given)$

$n_{yd} = K_{sz}\ \ K_{\sigma a}\ \ n_{ya}$ ... E(5.12)

$= 0.85 \times 1 \times 2.25 = 1.9125$

Assuming that there is no stress concentration, $K_{\sigma a} = 1$

Therefore, $\sigma_{yd} = \dfrac{300}{1.9125} = 156.86\, N/mm^2$

Substituting the values in Soderberg's equation, we get

$$\left(\frac{\frac{2625}{b}}{109.8}\right) + \left(\frac{\frac{1124.996}{b}}{156.86}\right) = 15mm$$

$$b = 31.07mm$$

Therefore, *Width of beam required = 31.07mm.*

Example 3.13: A round rod of diameter 1.2d is reduced to a diameter d with a fillet radius of 0.1d. This rod is to sustain a twisting moment that fluctuates between +2.5kNm and 1.5kNm together with a bending moment that fluctuates between +1kN-m and -1kNm. The rod is made of carbon steel 40C8. Determine a suitable value for d.

(VTU-Jan/Feb. 2003)

Solution:

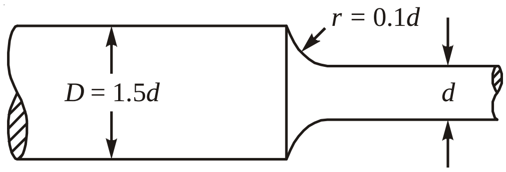

Figure-3.15

For the given *stepped rod,* $D = 1.2d,\ r = 0.1d,\ d = d$

Therefore, $\frac{r}{d} = \frac{0.1d}{d} = 0.1$

And $\frac{D}{d} = \frac{1.2d}{d} = 1.2$

Considering torsion, corresponding to $\frac{r}{d} = 0.1,\ \frac{D}{d} = 1.2,\ K_\tau = 1.33$ from figure-1.19A MDH

Similarly considering bending, corresponding to $\frac{r}{d} = 0.1,\ \frac{D}{d} = 1.2$ and referring figure-4.21A. *Stress concentration factor* $K_\sigma = 1.625$

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(M'_{bm}\right)^2 + \left(M'_{tm}\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad \text{... E(14.21)}$$

Where $M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{-1d}} M_{ba}$ ... E(14.22a)

$$M_{bm} = \frac{M_{b\max} + M_{b\min}}{2}$$

$$M_{b\max} = 1\text{kN-m}$$

$$M_{b\min} = (-)1\text{kN-m}$$

Therefore, $M_{bm} = \frac{1+(-1)}{2} = 0$

$$M_{ba} = \frac{M_{b\max} - M_{b\min}}{2}$$

$$= \frac{(1)-(-1)}{2} = 1\text{kN-m}$$

$$= 10^3 \times 10^3 \text{ N-mm}$$

$$= 10^6 \text{ N-mm}$$

$$\sigma_{sd} = \frac{\sigma_y}{factor\ of\ safety}$$

For 40C8 steel, $\sigma_{st} = 580 - 680 MPa\ say\ \sigma_{st} = 600 MPa$ ... T(1.6)

Assuming, $\sigma_y = 0.6\sigma_{st}$

$$= 0.6 \times 600 = 360 MPa$$

Assuming factor of safety = 3,

$$\sigma_{sd} = \frac{360}{2} = 120 MPa$$

To find σ_{-1d}, the design endurance stress

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

Where, $\sigma_{-1} = 0.5\sigma_u$

For 40C8 steel, $\sigma_u = 600 MPa = \sigma_{st}$

$$\sigma_{-1} = 300 MPa$$

$$n_{-1d} = K_{sz}\ K_{sr}\ K_l\ n_{-1a}$$

Use, $K_{sz} = 0.85$

$$K_{sr} = \frac{1}{e_{sr}} = \frac{1}{0.82}$$

Note: e_{sr} is obtained from figure-5.3 MDH, for $\sigma_u = 600 MPa$

$$K_l = 1.0 \ \textit{for bending}, \quad n_{-1a} = 3(\textit{given})$$

Therefore, $$n_{-1d} = 0.85 \times \frac{1}{0.82} \times 1 \times 3 = 3.11$$

Therefore, $$\sigma_{-1d} = \frac{300}{3.11} = 96.46 \, N/mm^2$$

Substituting we get,

$$M'_{bm} = 0 + \frac{120}{96.46} \times 10^6$$

$$= 1.244 \times 10^6 \text{ N-mm}$$

$$M'_{bm} = M_{tm} + \frac{\tau_{sd}}{\tau_{-1d}} M_{ta} \qquad \text{... E(14.22b)}$$

Where, $$M_{tm} = \frac{M_{t\max} + M_{t\min}}{2} \qquad \text{... E(5.56)}$$

$$= \frac{2.5 \times 10^6 + 1.5 \times 10^6}{2} = 2 \times 10^6 \text{ N-mm}$$

$$M_{ta} = \frac{M_{t\max} - M_{t\min}}{2} \qquad \text{... E(5.55)}$$

$$= \frac{2.5 \times 10^6 - 1.5 \times 10^6}{2} = 0.5 \times 10^6 \text{ N-mm}$$

$$\tau_{sd} = \frac{\tau_y}{\textit{factor of safety}}$$

$$= \frac{0.6 \times 200}{4.144} = 28.96 \, N/mm^2$$

Where, factor of safety,

$$n_{yd} = K_{sz} \ K_{\sigma a} \ n_{ya}$$

$$= 0.85 \times 1.625 \times 3 = 4.144$$

$$\tau_{-1d} = \frac{\tau_{-1}}{n_{-1d}}$$

$$= \frac{0.25 \times \sigma_u}{K_{sz} \;\; K_{\sigma a} \;\; n_{ya}}$$

$$= \frac{0.25 \times 600}{0.85 \times \frac{1}{0.84} \times 0.6 \times 1.625} = 152.04\, N/mm^2$$

$$e'_{sr} = 0.425 + 0.575 e_{sr} \qquad \text{... E(5.8)}$$

For [Figure-5.3 MDH], for $\sigma_u = 600 MPa$ and for *rough finished rod,*

$$e'_{sr} = 0.91$$

i.e., $0.91 = 0.425 + 0.575 e_{sr}$

Therefore, $e_{sr} = 0.84$

Therefore, $M'_{tm} = 2 \times 10^6 + \frac{28.96}{30.4} \times 0.5 \times 10^6$

$$= 2.4763 \times 10^6 \text{ N-mm}$$

Now, $$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(M'_{bm} \right)^2 + \left(M'_{tm} \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14.21)}$$

For 40C8 steel, $\sigma_{st} = 600 MPa$

Assuming $\sigma_y = 0.6 \sigma_{st}$

$$= 0.6 \times 600 = 360 MPa$$

$$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{0.5 \times 360}{factor\ of\ safety}$$

$$= \frac{180}{3} = 60$$

Therefore, $$D = \left[\frac{16}{\pi \times 60} \left\{ \left(2.4763 \times 10^6\right)^2 + \left(1.244 \times 10^6\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$D = 61.32$$

Use $D = 65mm$

Example 3.14: A round rod of diameter 1.2d has a semicircular groove of diameter 0.2d. This rod is to sustain a twisting moment that fluctuates between 2.5kNm and 1.5kNm together with a bending moment that fluctuates between +2kNm and -1kNm. Selecting carbon steel as material for the rod and choosing a factor of safety = 2.5, determine the safe value for d. (VTU-July 2006)

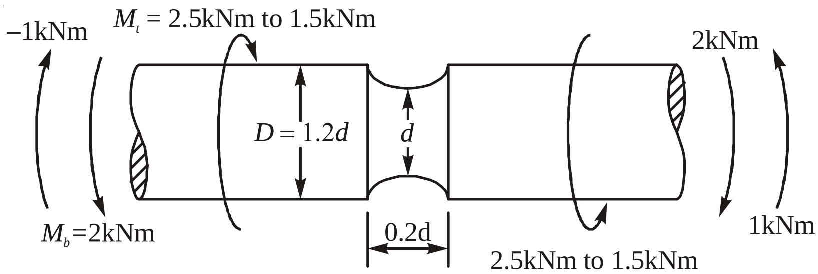

Figure-3.16

Solution:

The shaft is subjected to combined fluctuating bending and torsional loads. The diameter of shaft is given by,

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(M'_{bm}\right)^2 + \left(M'_{tm}\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14.21)}$$

To calculate τ_{ed}

For C30 steel (i.e., 30C8 steel), $\sigma_{ut} = 490 - 588\, N/mm^2$ ... T(1-25)

Say $\sigma_{ut} = 550MPa\left(an\ average\ value\right)$

Assuming $\sigma_y = 0.6\sigma_{ut}$

$$\sigma_y = 0.6 \times 550 = 330MPa$$

$$\tau_y = \frac{\sigma_y}{2} = \frac{330}{2} = 165MPa$$

$$\tau_{ed} = \frac{\tau}{factor\ of\ safety}$$

$$= \frac{165}{2.5} = 66 MPa$$

To calculate M'_{bm}

$$M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{yd}} \times M_{ba}$$

Where $M_{bm} = \dfrac{M_{b\max} + M_{b\min}}{2}$

$$M_{b\max} = 2\text{kNm}; \quad M_{b\min} = -1\text{kNm}$$

Therefore, $M_{bm} = \dfrac{2+(-1)}{2} = 0.5\,\text{kNm} = 5\times10^5\ \text{N-mm}$

Similarly, $M_{ba} = \dfrac{M_{b\max} - M_{b\min}}{2}$

$$= \frac{2-(-1)}{2} = 1.5\,\text{kNm} = 15\times10^5\ \text{N-mm}$$

$$\sigma_{sd} = \frac{\sigma_y}{factor\ of\ safety}$$

$$= \frac{330}{2.5} = 132 MPa$$

$$\sigma_{yd} = \frac{\sigma_y}{n_{yd}};$$

Where $n_{yd} = K_{sz}\ \ K_{\sigma a}\ \ n_{ya}$

$$K_{sz} = 0.85$$

For the given shaft,

$$\frac{D}{d} = \frac{1.2d}{d} = 1.2$$

$$\frac{r}{d} = \frac{0.1d}{d} = 0.1$$

Corresponding to $\frac{D}{d} = 1.2$ and $\frac{r}{d} = 0.1$,

$K_\sigma = 1.872$ for bending,

$K_\tau = 1.45$ for torsion.

Therefore, $n_{yd} = 0.85 \times 1.875 \times 2.5 = 3.9844$

Therefore, $\sigma_{yd} = \frac{330}{3.9844} = 82.82 MPa$

Therefore, $M'_{bm} = 5 \times 10^5 + \frac{82.82}{82.82} \times 15 \times 10^5$

$= 20 \times 10^5$ N-mm

$$M'_{tm} = M_{tm} + \frac{\tau_{sd}}{\tau_{yd}} \times M_{ta}$$

Where, $M_{tm} = \frac{M_{t\max} + M_{t\min}}{2}$

$$= \frac{25 \times 10^5 + 15 \times 10^5}{2} = 20 \times 10^5 \text{ N-mm}$$

$$M_{ta} = \frac{M_{t\max} - M_{t\min}}{2}$$

$$= \frac{25 \times 10^5 - 15 \times 10^5}{2} = 5 \times 10^5 \text{ N-mm}$$

$$\tau_{sd} = \frac{\tau_y}{factor\ \ of\ \ safety}$$

$$= \frac{0.6\sigma_y}{2.5} = \frac{0.6 \times 330}{2} = 79.2 MPa$$

$$\tau_{yd} = \frac{\tau_y}{n_{yd}}$$

$$= \frac{0.6_y}{K_{sz} \ K_{\sigma a} \ n_{ya}}$$

$$= \frac{0.6 \times 330}{0.85 \times 1.45 \times 2.5} = 64.26 MPa$$

Therefore, $$M'_{tm} = 20 \times 10^5 + \frac{79.2}{64.26} \times 5 \times 10^5$$

$$= 26.162 \times 10^5 \text{ N-mm}$$

Substituting in E(14-21) we get,

$$D = \left[\frac{16}{\pi \times 66} \left\{ \left(20 \times 10^5\right)^2 + \left(26.162 \times 10^5\right)^2 \right\} \right]^{\frac{1}{3}}$$

$$D = 63.33 mm$$

Adopt $d = 62mm$ (standard size)

Example 3.15: A stepped shaft of circular cross section shown in figure-3.17 is made of 20Mn steel $\left(\sigma_y = 431.5MPa\right)$***. Determine the value of d and the fillet radius r so that the maximum stress will be limited to a ratio corresponding to a factor of safety of 2.5, and taking stress concentration factor into account.*** ***(VTU-Dec 07/Jan 08)***

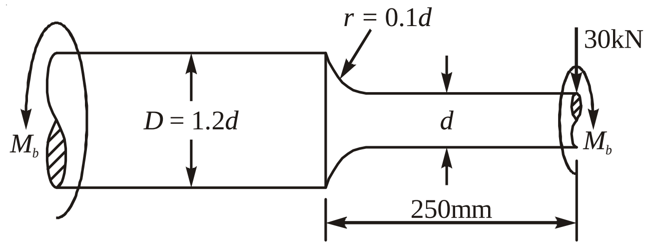

Figure-3.17

Solution:

The stepped shaft is subjected to bending and hence the bending stress induced is given by

$$\sigma_b = \frac{M_b}{Z_b}$$

Where, $$M_b = 30\text{kN} \times 250$$

$$= 30 \times 10^3 \times 250 = 75 \times 10^5 \text{ N-mm}$$

To calculate stress concentration factor K_σ

For the given shaft,

$$\frac{D}{d} = \frac{1.2d}{d} = 1.2$$

$$\frac{r}{d} = \frac{0.1d}{d} = 0.1$$

Referring to figure 4.21A,

$$K_\sigma = 1.66$$

Stress concentration factor,

$$K_\sigma = \frac{\sigma_{b\max}}{\sigma_{bnominal}}$$

Where, $K_\sigma = 1.66$

$$\sigma_{b\max} = \frac{\sigma_y}{factor\ of\ safety}$$

$$= \frac{431.5}{2.5} = 172.6MPa$$

Therefore, $K_\sigma = \frac{\sigma_{b\max}}{\sigma_{bnominal}}$

$$1.66 = \frac{172.6}{\sigma_{bnominal}}$$

Therefore, $\sigma_{bnominal} = 103.98MPa$

But, $\sigma_{bnominal} = \frac{M_b}{Z_{b\min}}$

$$= \frac{75 \times 10^5}{\frac{\pi d^3}{32}} = 103.98$$

Therefore $d = 90.23mm$ *and* $r = 0.1d = 9.023mm$