Chapter 5

DESIGN OF SHAFTS

Design of *shaft* consists of determination of *shaft dimensions to sustain* and to *transmit the given power.* Shafts ma be solid or hollow.

Power from one shaft to the other is transmitted through *sprockets (using chains), Gears or Pulleys (using belts). Maximum shear stress theory is used for ductile materials and Maximum normal stress theory for brittle materials* wile designing shafts.

In most of the engineering applications, the shaft materials are ductile and are subjected to combined *bending* and *torsional loads.* The torque M_t transmitted by the shaft is determined based on power transmitted. The vertical and horizontal loads on members mounted on shaft are computed and the corresponding bending moment diagrams due to vertical and horizontal loads are drawn. From these diagrams the resultant bending moment diagram is drawn and hence the maximum bending moment M_b on the shaft is obtained.

The values of M_b and M_t are multiplied by *shock* and *endurance factors* K_b and K_t obtained from table (14.2)

The following are the important types of problems that are covered under this chapter *"Design of Shafts"*.

i) Shafts supported at one end in bearing and loaded at the free end (Cantilever).

ii) Shafts simply supported between bearings at the ends and pulley or gears keyed to it between bearings.

iii) Shafts carrying pulleys/gears in between bearings and pulley/gears overhanging one of the bearings.

iv) Shafts with pulley or gears overhanging both bearing.

As mentioned earlier, power transmission shafts carry pulley or gears keyed to then and hence are subjected to bending. Therefore, the knowledge of load and bending moment diagrams are essential for the *Designer,* while designing shafts.

Design Formulae used

a) Solid Shafts with Dynamic Loads

i) The diameter D of the shaft subjected to torsion is given by,

$$D = \left[\frac{16}{\pi \tau_{ed}} \left(K_t M_t \right) \right]^{\frac{1}{3}} \qquad \text{... E(14-9)}$$

ii) The diameter D of the shaft subjected to bending is given by,

$$D = \left[\frac{32}{\pi \sigma_{ed}} \left(K_b M_b \right) \right]^{\frac{1}{3}} \qquad \text{... E(14-10)}$$

Where, $D = Shaft\ diameter;\ mm$

$\sigma_{ed} = Design\ normal\ stress;\ MPa$

$M_t = Torque\ transmitted;$ N-mm

$M_b = Bending\ moment;$ N-mm

Note: Torque $M_t = \frac{9550N}{n}$ N-m

Where, $N = Power\ transmitted\ in\ kW$

$n = Speed\ of\ shaft\ in\ rpm$

K_b and K_t are shock and endurance factors and are obtained from Table (14.2)

iii) The diameter D of the shaft subjected to combined bending and torsion (According to maximum shear stress theory) is given by,

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

iv) The diameter of the shaft based on torsional rigidity is given by,

$$D = \left[\frac{584 M_t L}{G\theta} \right]^{\frac{1}{4}} \qquad \text{... E(14-17)}$$

Where, $L = Length\ of\ the\ shaft;\ mm$

$G = Modulus\ of\ rigidity\ of\ the\ material\ of\ shaft;\ MPa$

$\theta = Angular\ deflection\ of\ shaft;\ Degrees$

b) Hollow Shafts with Dynamic Loads

i) The outside diameter D_0 subjected to simple torsion is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} K_t M_t \right]^{\frac{1}{3}} \qquad \text{... E(14-34)}$$

Where, $k = \dfrac{D_i}{D_0}$; *Ratio of inside diameter D_i to outside diameter D_0*

ii) The outside diameter of shaft D_0 subjected to simple bending is given by

$$D_0 = \left[\frac{32}{\pi \sigma_{ed} \left(1 - k^4\right)} K_b M_b \right]^{\frac{1}{3}} \qquad \text{... E(14-35)}$$

iii) The outside diameter D_0 of the shaft subjected to combined bending and torsion according to maximum shear stress theory is given by,

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-37)}$$

iv) The outside diameter D_0 based on torsional rigidity is given by

$$D_0 = \left[\frac{584 M_t L}{\left(1 - k^4\right) G \theta} \right]^{\frac{1}{4}} \qquad \text{... E(14-42)}$$

c) Solid Shafts with Fluctuating Loads

i) The diameter of solid shaft subjected to fluctuating torsion is given by

$$D = \left[\frac{16}{\pi} \left(\frac{M_{tm}}{\tau_{yd}} + \frac{M_{ta}}{\tau_{-1d}} \right) \right]^{\frac{1}{3}} \qquad \text{... E(14-18)}$$

Where, $M_{tm} = \textit{Mean torque} = \dfrac{M_{t\max} + M_{t\min}}{2}$

$M_{ta} = \textit{Torque amplitude} = \dfrac{M_{t\max} - M_{t\min}}{2}$

$$\tau_{yd} = Design\ yield\ shear\ stress = \frac{\tau_y}{n_{yd}}$$

$$\tau_{-1d} = Design\ endurance\ shear\ stress = \frac{\tau_{-1}}{n_{-1d}}$$

Where, $\tau_{-1} = Endurance\ shear\ stress = 0.25\sigma_u\ (assumption)$

ii) The diameter of the shaft subjected to fluctuating bending load is given by

$$D = \left[\frac{32}{\pi}\left(\frac{M_{bm}}{\sigma_{yd}} + \frac{M_{ba}}{\sigma_{-1d}}\right)\right]^{\frac{1}{3}} \qquad \text{... E(14-19)}$$

Where, $M_{ba} = Bending\ moment\ amplitude = \frac{M_{b\max} - M_{b\min}}{2}$

$$M_{bm} = Mean\ bending\ moment = \frac{M_{b\max} + M_{b\min}}{2}$$

$$\sigma_{yd} = Design\ yield\ stress = \frac{\sigma_y}{factor\ of\ safety}$$

Where, $\sigma_y = Yield\ stress;\ MPa$

$$\sigma_{-1d} = Design\ endurance\ stress = \frac{\sigma_{-1}}{factor\ of\ safety}$$

$$\sigma_{-1} = Endurance\ stress;\ MPa$$

iii) The diameter of the shaft subjected to combined fluctuating bending and torsional loads according to maximum shear stress theory is given by

$$D = \left[\frac{16}{\pi\tau_{ed}}\left(M'^2_{bm} + M'^2_{tm}\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14-21)}$$

Where, $M'_{bm} = M_{bm} + \frac{\sigma_{sd}}{\sigma_{-1d}} M_{ba}$ ... E(14.22a)

$$M'_{tm} = M_{tm} + \frac{\tau_{sd}}{\tau_{-1d}} M_{ta} \qquad \text{... E(14.22b)}$$

and $\sigma_{sd} = \dfrac{\sigma_y}{factor\ of\ safety}$

$$\sigma_{-1d} = \frac{\sigma_{-1}}{factor\ of\ safety}$$

$$\tau_{sd} = \frac{\tau_y}{factor\ of\ safety} = \frac{0.6\sigma_y}{factor\ of\ safety}$$

$$\tau_{-1d} = \frac{\tau_{-1}}{factor\ of\ safety} = \frac{0.25\sigma_u}{factor\ of\ safety}$$

Design procedure for shafts

The following general procedure is followed wile designing shafts

Step I: Selection of suitable equation

Study the problem and determine the type of load while designing shafts

a) If the shaft is subjected to only *torsional loads,* use equation (14-9) to determine the shaft diameter *'D'*.

b) If the shaft is subjected to only *bending loads,* use equation (14-10) to determine the shaft diameter.

c) If the shaft is subjected to combined *bending and torsional loads,* use equation (14-11) or equation (14-12).

Equation (14-11) is used if *maximum-normal stress theory* is employed.

Equation (14-12) is used if *maximum-shear stress theory* is used (*Generally this theory is employed and it gives a safe value of shaft diameter*).

d) If the *angle of twist* of shaft is to be restricted to a predefined value, use

Equation (14-17) for *solid shafts, and*

Equation (14-42) for *hollow shafts.*

Step II: Calculation of torque M_t and Bending moment M_b

a) Calculation of torque M_t

M_t is obtained using equation,

$$M_t = 9550\frac{N}{n} N\text{ - }metre \qquad \text{... E(19.3c)}$$

Where, $N = Power\ transmitted\ in\ kW$

$n = Speed\ of\ rotation\ of\ shaft\ in\ rpm$

Use equation (14-36) for hollow shafts if design is based on *maximum normal stress theory.*

Use equation (14-37) for hollow shafts if design is based on *maximum shear stress theory.*

For hollow shafts, use equation (14-34) when subjected on *only torsion*

Use equation (14-35) for shafts subjected to *only bending loads.*

Note: If power is transmitted through pulleys, then

Torque $M_t = (T_1 - T_2) r_p$ N-m

Where, $T_1 \text{ and } T_2$ *are belt tensions in N*, $T_1 > T_2$

$r_p = \text{Radius of pulley; metre}$

Also, ratio of belt tension for drives using flat pulleys is given by

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Ratio for drives using V-belts and ropes is given by

$$\frac{T_1}{T_2} = e^{\frac{\mu\theta}{e^{\sin\alpha}}}$$

Where $\mu = \text{Coefficient of friction between pulleys and belt}$

$\theta = \text{Angle of wrap or angle of lap or angle of contact in radians}$

$2\alpha = \text{Groove angle of V - pulleys}$

1) If the power is transmitted through pulleys of equal size, then the angle of lap $= 180^0 = \pi$ radians.

2) If power is transmitted through gears between shafts, then torque $M_t = F_r r$

Where $F_t = \text{Tangential force on Gear/Pinion mounted on shaft}$

$r = \text{Radius of Gear/Pinion}$

3) M_b is determined by calculating the loads on members (Gears and Pulleys) mounted on the shaft.

Step III: Calculation of loads on pulley

1) If the pulley is receiving / transmitting power from / to another pulley mounted

behind it then, the total horizontal load on pulley mounted on shaft $= (T_1 + T_2)$. The vertical load on pulley $= W_p =$ weight of pulley.

2) If the pulley is receiving / transmitting power from / to another pulley mounted exactly below it, then *horizontal load* on pulley = zero.

3) If the pulley is receiving / transmitting power from / to another pulley mounted behind it through an inclined belt drive inclined at an angle θ^0 to the horizontal (sloping upwards) then, the total tension $(T_1 + T_2)$ is resolved in the horizontal and vertical directions as shown in figure-5.1

Figure-5.1

$Total\ horizontal\ load = (T_1 + T_2)\cos\theta$

$Total\ vertical\ load = Difference\ of\ (T_1 + T_2)\sin\theta\ \ and\ \ W_p$

Note: If $(T_1 + T_2)\sin\theta > W_p$, then vertical load of $(T_1 + T_2)\sin\theta - W_p$, *acts upwards* $\uparrow$

If $(T_1 + T_2)\sin\theta < W_p$, then vertical load of $W_p - (T_1 + T_2)\sin\theta$, *acts downwards* $\downarrow$

4) If the pulley is receiving / transmitting power from / to another pulley mounted behind it through a belt inclined at an angle θ with the horizontal (sloping downwards) then

The horizontal load on pulley $= (T_1 + T_2)\cos\theta$

Vertical load on pulley $= (T_1 + T_2)\sin\theta + W_p \downarrow$

Figure-5.2

Step IV: Calculation of loads on Gear / Pinion

i) If pinion is mounted on the shaft, and tangential force on this is acting downwards, then

Figure-5.3

The vertical load on pinion is

$$F_t = \frac{M_t}{r_p} \downarrow$$

Where, $r_p = Radius\ of\ \ pinion$

$$= \frac{d_p}{2}$$

Where, $d_p = Pitch\ circle\ diameter\ of\ Pinion$

$$d_p = mz_p$$

$$m = Module, mm$$

$$z_p = Number\ of\ teeth\ on\ Pinion$$

ii) If the tangential force on Pinion acts upwards, then vertical load on Pinion is

$$F_t = \frac{M_t}{r_p} \uparrow$$

iii) The horizontal load on Pinion $= F_r =$ separating force (or radial force) given by

$$F_r = F_t \tan \beta$$

Where, $\beta =$ Pressure angle of Gears (Standard pressure angles are 14 ½^0 and 20^0)

F_t acts on Pinion along the *line of centres* of Gear and Pinion.

iv) Unless otherwise specified, the weights of Gears and Pinions are neglected.

v) If Gear is mounted below the Pinion, then the horizontal load on Pinion $F_t = \frac{M_t}{r_p}$ and vertical load on Pinion $= F_r = F_t \tan \beta$

Figure-5.4

vi) If Gear is mounted above Pinion, then

Horizontal load on Pinion $F_t = \frac{M_t}{r_p}$ and

Vertical load on Pinion $F_r = F_t \tan \beta$

Where $\beta = Pressure\ angle\ of\ Gears = 14\tfrac{1}{2}^0\ or\ 20^0$

Step V: Direction of rotation of pulleys

Figure-5.5

If the slack side is at the top, then Pulley rotates in the *clockwise direction* and *vice versa.*

Step VI: Direction of rotation of Gears/Pinion

Figure-5.6

If Pinion is mounted on shaft and the tangential force acts in the *downward direction*, then the Pinion rotates in the *counter clockwise direction. Refer figure-5.6*

Figure-5.7

If Pinion is mounted on shaft and the tangential force on it acts upwards, then the direction of rotation of Pinion is clockwise as shown in figure-5.7

Note: It is to be noted that the Pinion and Gear are shown without meshing to get direction and rotation of Gear or Pinion.

Step VII: Bending Moment Diagrams

Knowing the values of loads at the Pulleys and Gears, the horizontal and vertical bending moment diagrams are drawn separately and the value of resultant bending moment M_b is obtained.

Load and Bending moment Diagrams

In this portion, generally cases of bending moment diagrams are shown

a) A pulley of weight W_p mounted on shaft at its middle and receiving power from another pulley mounted on a parallel shaft behind it.

Figure-5.8

b) A cantilever shaft with pulley at the free end

Figure-5.9

c) A shaft mounted between bearings carrying two unequal size pulleys of weights W_1 and W_2

Figure-5.10

d) Shaft mounted between bearings carrying a pulley inside the bearings and a pinion overhanging the right hand bearing.

Figure-5.11

e) A shaft between bearings carrying a pulley and a pinion on the extension of shaft

Figure-5.12

Step VIII: Selection of shock and endurance factors

The *shock and fatigue factors* K_b and K_t are obtained from Table (14.2)

For *Steady loads,* $K_t = 1, K_b = 1.5$ *(When pulley or pulleys are present)*

$$K_t = 1.25 \ \ and \ \ K_b = 1.5 \ to \ 2$$

For *suddenly applied load with heavy shock, (When shaft carries only Gears)*

$$K_t = 1.5 \ to \ 3 \ \ and \ \ K_b = 2 \ to \ 3$$

Step IX: Selection of material of shaft

Generally carbon steels such as 40C or 45C8 (C40 or C45 steel) is selected for the material of the shaft. Knowing the value of σ_y, yield shear stress τ_y is approximated as $0.5\sigma_y$. The design shear stress τ_{ed} is obtained by using suitable factor of safety.

Step X: Calculation of size of shaft

The value of shaft diameter *D* obtained from the above information is rounded off to the nearest next standard size.

Example 5.1: A commercial steel shaft 1.5m long supported between bearings carries a pulley of diameter 600mm weighing 1000N at its centre and transmitting 25kW at 200rpm to another pulley mounted behind the shaft by a horizontal belt drive. Taking ration of belt tensions as 3, determine

i) The diameter of the shaft required.

ii) The angle of twist of shaft between one of the bearings and the pulley taking $G = 8\times10^3 \ N/mm^2$.

Solution:

a) The shaft is subjected to combined bending and torsion and hence the shaft diameter D is given by

$$D = \left[\frac{16}{\pi\tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \quad \text{... E(14-12)}$$

Where, $K_b \ and \ K_t$ are shock and endurance factors. Assuming stead loading,

$$K_b = 1.5 \ \ and \ \ K_t = 1 \quad \text{... T(14-2)}$$

$$M_t = Torque \ transmitted = \frac{9550N}{n} \ \text{N-m}$$

$$= \frac{9550 \times 25 \times 10^3}{200} = 1193750 \text{ N-mm}$$

Note: For commercial shaft, according to ASME code, which uses maximum shear stress theory, the value of M_b *and* M_t are to be multiplied by combined shock and fatigue factors. $\tau_{ed} =$ allowable shear stress $= 40\, N/mm^2$ for shaft with keyway and for shaft without keyway, $\tau_{ed} = 55\, N/mm^2$.

I) To find M_b, the maximum bending moment

The total vertical load on Pulley $= W_p = Weight\ of\ pulley = 1000N \downarrow$

The total horizontal load on Pulley $= (T_1 + T_2)$

i) To find T_1 *and* T_2, the belt tensions

Given, Ratio of belt tensions

$$\frac{T_1}{T_2} = 3$$

Therefore, $T_1 = 3T_2$ ... (1)

Also, Torque $M_t = (T_1 - T_2) r$

Where $r = Radius\ of\ pulley\ in\ mm$

Therefore, $1193750 = (3T_2 - T_2)\frac{600}{2}$

i.e., $T_2 = 1989.58N$

and $T_1 = 3T_2$

$= 3 \times 1989.58 = 5968.74N$

Therefore, the total horizontal load, $(T_1 + T_2) = 7958.32N$

II) To draw the load and bending moment diagrams

To find M_b, the maximum bending moment

Figure-5.13

a) Vertical load diagrams (VLD)

Mark line AB indicating the shaft supported between bearings and mark at its centre the vertical load $W_p = 1000N$.

Let R_{AV} and R_{BV} be the reactions at bearings A and B due to *vertical load only.*

Then, $R_{AV} + R_{BV} = 1000$

Because of symmetry, $R_{AV} = R_{BV}$

Therefore, $R_{AV} = R_{BV} = \dfrac{1000}{2} = 500N$

b) Vertical bending moment diagram (VBMD)

Bending moment is zero at A and B and is maximum at the centre of shaft and is given by

$$M_{bv} = R_{AV} \times 750 \;\; or \;\; R_{BV} \times 750$$
$$= 500 \times 750 \text{ N-mm}$$
$$= 375000 \text{ N-mm}$$

c) Horizontal load diagram (HLD)

Total horizontal load $(T_1 + T_2) = 7958.32N$

Note: By convention horizontal load is shown downward at the pulley if acting to right and upwards if acting to left. The vice versa is also valid.

Let R_{AH} and R_{BH} be the reactions A and B due to horizontal load $(T_1 + T_2)$

Then, because symmetry,

$$R_{AH} = R_{BH} = \left(\frac{T_1 + T_2}{2}\right)\downarrow$$

$$= \frac{7958.32}{2} = 3973.16N$$

d) Horizontal bending moment diagram (HDBM)

Bending moment is *zero* at the bearings A and B and is *maximum* at the centre of shaft given by,

$$M_{BH} = R_{AH} \times 750$$
$$= 3979.16 \times 750 = 2984370 \text{ N-mm}$$

The maximum bending moment (resultant bending moment) at the pulley is

$$M_b = \sqrt{M_{bv}^2 + M_{bh}^2}$$
$$= \sqrt{(375000)^2 + (2984370)^2}$$

$$= 3003373.14 \text{ N-mm}$$

Therefore, $$D = \left[\frac{16}{\pi \times 40} \left\{ (1.5 \times 3007838)^2 + (1 \times 1193750)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 84.07 mm$$

Adopt $D = 85 mm \, (Standard\ size)$

b) To find the angle of twist θ

For a solid shaft

$$\theta = \frac{584 M_t L}{G D^4} \qquad \text{... E(14-17)}$$

Where, $L = Length\ between\ bearings\ and\ the\ pulley = 750 mm$

$D = 85 mm$

$G = 80 \times 10^3\ N/mm^2$

Therefore, $$\theta = \frac{584 \times 1193750 \times 750}{80 \times 10^3 (85)^4} = 0.1252^0$$

Example 5.2: A 45C8 (C45 steel) shaft is required to transmit 25kW at 1000rpm. Determine the shaft diameter based on a factor of safety of 3.

Solution:

The shaft is subjected to torsion and the diameter of shaft subjected to only torsion is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} (K_t M_t) \right]^{\frac{1}{3}} \qquad \text{... E(14-9)}$$

Where, $K_t = 1$

$$M_t = \frac{9550 \times 25 \times 10^3}{1000} \text{ N-mm}$$

$$= 238780 \text{ N-mm}$$

For 45C8 steel, $\sigma_u = 618 MPa$ ... T(1-25)

Assuming $\sigma_y = 0.6\sigma_u$

$= 0.6 \times 618 = 370.8 \, N/mm^2$

$\tau_y = 0.5\sigma_y$

$= 0.5 \times 370.8 = 185.4 \, N/mm^2$

$$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{185.4}{3} = 61.8 \ N/mm^2$$

Therefore, $D = \left[\frac{16}{\pi \times 61.8} \times 1 \times 238750 \right]^{\frac{1}{3}}$

$= 26.99 mm$

Use standards size $D = 30mm$

Example 5.3: A hollow shaft 1000mm long supported between bearings is subjected to a maximum bending moment of 100N-m and a torque of 150N-m. Incidentally, the angle of twist in the shaft is to be restricted to 2^0. Taking $G = 80 \times 10^3 \ N/mm^2$, Permissible shear stress = 55MPa, inside diameter of shaft = 0.6 times outside diameter, determine the dimensions of the hollow shaft.

Solution:

The shaft is subjected to combined bending and torsion and hence the outside diameter of hollow shaft subjected to combined bending and torsion is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \quad . \ . \ .$$

E(14-37)

Where $\tau_{ed} = 55 \, N/mm^2 \left(Given\right)$

$K_b = 1.5$

$K_t = 1\left(Assuming\ steady\ load\right)$ ... T(14-2)

$M_b = 150$ N-m

$$= 150 \times 10^3 \text{ N-mm}$$

$$M_t = 100 \text{ N-m}$$

$$= 100 \times 10^3 \text{ N-mm}$$

$$k = \frac{D_i}{D_0} = Ratio\ of\ diameters = 0.6$$

$$D_0 = \left[\frac{16}{\pi \times 55\left(1 - 0.6^4\right)} \left\{ \left(1.5 \times 150 \times 10^3\right)^2 + \left(1 \times 100 \times 10^3\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 29.7mm = 30mm$$

Therefore, $D_i = 0.6 D_0$

$$= 0.6 \times 30 = 18mm$$

Based on torsional rigidity

$$D_0 = \left[\frac{584 M_t L}{G\theta\left(1 - k^4\right)} \right]^{\frac{1}{4}} \qquad \text{... E(14-42)}$$

$$= \left[\frac{584 \times 100 \times 10^3 \times 1000}{80 \times 10^3 \times 2\left(1 - 0.6^4\right)} \right]^{\frac{1}{4}}$$

$$= 25.45mm$$

Therefore, $D_i = 0.6 D_0$

$$= 0.6 \times 25.45 = 15.27mm$$

Note: Adopt higher of the two values of D_0 and the corresponding value of D_i

Hence $D_0 = 30mm$

Therefore, $D_i = 0.6 \times D_0$

$$= 0.6 \times 30 = 18mm$$

Therefore, the dimensions of hollow shaft are,

Outside diameter $D_0 = 30mm$

Inside diameter $D_i = 18mm$

Example 5.4: A commercial steel shaft 1m long supported between bearings has a pulley of diameter 600mm weighing 1kN driven by a horizontal belt keyed to the shaft at a distance of 400mm to the left of the right bearing and receives 25kW at 1000 rpm. Power from the shaft is transmitted from a 20^0 spur pinion of pitch circle diameter 200mm which is mounted 200mm to the right of the left bearing to a gear such that the tangential force on gear acts vertically upwards. Taking the ration of belt tension = 3. Determine the standard size of the shaft required based on maximum shear stress theory.

Solution:

The shaft is subjected to combined bending and torsion. (Note: Bending due to weight of pulley, belt tension and gear force, torsion due to power transmitted). Hence the diameter of the shaft is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $\tau_{ed} = 40\, N/mm^2$ (for commercial shaft with keyway as per ASME code)

$K_b = 1.75$

$K_t = 1.25$ (assuming suddenly applied load with minor shock, since Gear is present)

$$M_t = \frac{9550 \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 25 \times 10^3}{1000} = 238750 \text{ N-mm}$$

To find the maximum bending moment M_b

Consider the pulley

Total horizontal load $THL = \left(T_1 + T_2 \right)$

To find T_1, T_2

The ration of belt tensions

$$\frac{T_1}{T_2} = e^{\mu\theta} = 3$$

Therefore, $T_1 = 3T_2$ ... (1)

Also $M_t = (T_1 - T_2) r_p$

Where $r_p = Radius\ of\ pulley$

$$238750 = (3T_2 - T_2)\frac{600}{2}$$

$$T_2 = 397.92N$$

$$T_1 = 1193.76N$$

$Total\ horizontal\ load\ \ THL = (T_1 + T_2) = 1591.68N \leftarrow$

$Total\ vertical\ load\ \ TVLW_p \downarrow = 1000N \downarrow$

Consider the pinion mounted on the shaft

$$TVL = F_t \downarrow$$

$$= \frac{M_t}{r_p}$$

$$= \frac{238750}{\frac{200}{2}} N = 2387.5N \downarrow$$

$$THL = F_r$$

$$= F_t \tan\beta \leftarrow$$

Where $F_r = Radial\ force$

$\beta = 20^0\ (given)$

$$\left[\beta = Pressure\ angle = 14\tfrac{1}{2}^0\ or\ 20^0\ (Standard\ values\ of\ pressure\ angle)\right]$$

$$F_r = F_t \tan\beta$$

$$= 2387.5 \tan 20 = 868.99N$$

Note: F_r acts normal to F_t along the line of centres of Gears.

Figure-5.14

a) Horizontal load diagram (HLD)
On the shaft, the horizontal loads on pinion and pulley act as shown.

b) Horizontal bending diagrams (HBMD)
Let R_{AH}, R_{BH} be the reactions at *A* and *B* due to horizontal loads only,

Then, $R_{AH} + R_{BH} = F_r + \left(T_1 + T_2\right)$

$= 868.99 + 1591.68$

$= 2460.67N$

Taking moments about *A* and equating *clockwise moments to anticlockwise moments,* we get

$$R_{BH} \times 1000 = 868.99 \times 200 + 1591.68 \times 600$$

Therefore, $R_{BH} = \dfrac{173798 + 955008}{1000}$

$= 1128.81N$

$R_{AH} = 2460.67N - 1128.81N$

$= 1331.86N$

Bending moments is zero at bearings *A* and *B*
At the Pinion, Bending moment

$= R_{AH} \times 200$

$= 1331.86 \times 200$

$= 266372$ N-mm

At the Pulley, Bending moment

$= R_{BH} \times 400$

$= 1128.81 \times 400$

$= 451524$ N-mm

Note: The value of M_b at Pinion and Pulley remain same in magnitude even if the horizontal loads are taken at the respective members in the upwards direction.

c) Vertical load diagrams (VLD)

Mark at Pinion and Pulley, the vertical loads $F_t \downarrow$ and weight of pulley $W_p \downarrow$

Let R_{AV} and R_{BV} be the reactions at A and B due to vertical loads only.

From figure-5.14,

$$\left(R_{AV} + R_{BV}\right) \uparrow = W_p \downarrow + F_t \downarrow$$

$$\left(Total\ upward\ load\right) = \left(Total\ downward\ load\right)$$

$$= 100 + 2387.5 = 3387.5N$$

Taking moments about A,

$$R_{BV} \times 1000 = F_t \times 200 + W_p \times 600$$

$$= 2387.5 \times 200 + 1000 \times 600$$

Therefore, $R_{BV} = 10773.5N$

$$R_{AV} = 3387.5 - 1077.5$$

$$= 2310N$$

d) Vertical bending moment diagram (VBDM)

Bending moments are zero at A and B

At the Pinion, Bending moment

$$= 2310 \times 200$$

$$= 462000N$$

Bending moment at the Pulley

$$= R_{BV} \times 4000$$

$$= 1077.5 \times 400$$

$$= 431000 \text{ N-mm}$$

Resultant BM at pinion

$$M_{b1} = \sqrt{\left(266372\right)^2 + \left(462000\right)^2}$$

$$= 533289.83 \text{ N-mm}$$

Resultant BM at pulley

$$M_{b2} = \sqrt{\left(451524\right)^2 + \left(431000\right)^2}$$

$$= 624207.44 \text{ N-mm}$$

Therefore, Maximum bending moment on shaft $= M_b = M_{b2}$

$= 624207.44\ \text{N-mm}\left(Higher\ of\ the\ two\ values\ M_{b1}\ and\ M_{b2}\right)$

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(K_b M_b\right)^2 + \left(K_t M_t\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

$$= \left[\frac{16}{\pi \times 40}\left\{\left(1.75 \times 624207.44\right)^2 + \left(1.25 \times 238750\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$= 52.44mm$$

Use shaft diameter $D = 55mm\left(Standard\ size\right)$

Example 5.5: A commercial steel shaft 1m long supported between bearings carries a pulley of diameter 600mm weighing 1kN located 400mm to the right of the right hand bearing and receives 25kW at 1000 rpm by a horizontal belt drive. The power from the shaft is transmitted by a spur pinion of 20^0 pressure angle having pitch circle diameter 200mm to a spur gear such that the tangential force on the gear acts vertically upwards. The pinion is keyed to the shaft at a distance of 200mm to the right of the left end bearing. Taking the ration of belt tensions as 3, determine the diameter of shaft required.

Solution:
The shaft is subjected to combined bending and torsional loads.
Therefore, the shaft diameter is given by

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(K_b M_b\right)^2 + \left(K_t M_t\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $\tau_{ed} = 40\ N/mm^2$ *(for commercial shaft with keyway according to ASME code)*

$K_b = 1.75$

$K_t = 1.25$ (Assuming suddenly applied load with minor shock)

$$M_t = \frac{9550N \times 10^3}{n}\ \text{N-mm}$$

$$= \frac{9550 \times 25 \times 10^3}{1000} = 238750 \text{ N-mm}$$

To find M_b, the maximum bending moment

Consider the pulley

The total horizontal load on the pulley $= (T_1 + T_2) \leftarrow$

Where, $\frac{T_1}{T_2} = \textit{Ratio of the belt tensions} = 3(\textit{given})$

i.e., $T_1 = 3T_2$

Also, *Torque* $M_t = (T_1 - T_2) r_p$

Where, $r_p = \textit{Radius of pulley} = \frac{600}{2} mm$

$$238750 = (3T_2 - T_2)\frac{600}{2}$$

Therefore, $T_2 = 397.92N$

And $T_1 = 397.92 \times 3 = 1193.76N$

Total horizontal load $= (T_1 + T_2)$

$= 397.92 + 1193.76 = 1591.68N \leftarrow$

Consider the Pinion

The total vertical load $= F_t \downarrow = \frac{M_t}{r_p} \downarrow$

$$= \frac{238750}{\frac{200}{2}} = 2387.5N \downarrow$$

The total horizontal load $= F_r = F_t \tan \beta \leftarrow$

$= 2387.5 \tan 20 = 868.98N \leftarrow$

a) Horizontal load diagram

Horizontal load on Pinion $= F_r = 868.95N \leftarrow$

Horizontal load on Pulley $= (T_1 + T_2)$

$= 1591.68 N \leftarrow$

Let R_{AH} and R_{BH} be the reactions at A and B due to horizontal loads only. Then,

$$R_{AH} + R_{BH} = 868.96 + 1591.68$$
$$= 2460.66 N$$

Now taking moments about A,

$$1591.68 \times 1400 + 868.96 \times 200 = R_{BH} \times 100$$

Therefore, $R_{BH} = 2402.15 N$

And $R_{AH} = 2460.66 - 2402.15 = 58.51 N$

b) Horizontal BMD

BM is zero at the bearing A and Pulley

(BM is 11702 N-mm at pinion and 63672 N-mm at bearing B)

Figure-5.15

c) Vertical load diagram

The vertical load at pinion is $F_t = 2387.5 N \downarrow$

The vertical load at pulley is $W_p = 1000 N \downarrow$

Let R_{AV} and R_{BV} be the reactions at A and B due to vertical loads only

Then, $R_{AV} + R_{BV} = F_t + 1000$

$$= 2387.5 + 1000$$
$$= 3387.5 N$$

Taking moments about A,

$$W_p \times 1400 + F_t \times 200 = R_{BV} \times 1000$$

$$1000 \times 1400 + 2387.5 \times 200 = R_{BV} \times 1000$$

Therefore, $R_{BV} = 1877.5 N$

And $R_{AV} = 1510 N$

d) Vertical BMD

BM is zero at bearing A and Pulley

BM at Pinion $= R_{AV} \times 200$

$= 1510 \times 200 = 302000$ N-mm

BM at Pinion B

$= W_p \times 400$

$= 100 \times 400 = 400000$ N-mm

Resultant bending moment (M_b)

At pinion, bending moment

$$M_{b1} = \sqrt{(11702)^2 + (3020000)^2}$$

$$= 302226.63 \text{ N-mm}$$

At the bearing, bending moment

$$M_{b2} = \sqrt{(636672)^2 + (400000)^2}$$

$$= 751898 \text{ N-mm}$$

Therefore, the maximum BM on the shaft i.e., M_b is M_{b2} (higher of the two value of M_b).

i.e., $M_{b2} = 751898$ N-mm

Therefore, $$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ (K_b M_b)^2 + (K_t M_t)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-2)}$$

$$= \left[\frac{16}{\pi \times 40} \left\{ (1.75 \times 751898.42)^2 + (1.25 \times 238750)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 55.59 mm$$

Therefore, Use shaft diameter $D = 60mm$ (Standard size).

Example 5.6: A commercial steel shaft 1000mm long supported between bearings carries a cast iron pulley of 600mm diameter weighing 1kN located 400mm to the right of the right bearing and receives 25kW at 1000 rpm from a horizontal belt drive. Power from the shaft is transmitted to a spur gear through a 20^0 spur pinion of pitch circle diameter 200mm overhanging the left bearing by 200mm such that the tangential force on the gear acts vertically upwards.

Taking the ration of belt tensions as 3, determine the size of the shaft required taking shock and fatigue factors $K_b = 1.75$ and $K_t = 1.25$

Solution:

The shaft is subjected to *combined bending and torsion.* Therefore the shaft diameter is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $\tau_{ed} = 40MPa$ (for commercial shaft with keyway as per ASME code).

$K_b = 1.75$

$K_t = 1.25$

Torque, $M_t = \dfrac{9550N \times 10^3}{n}$ N-mm

A) To find M_b, the maximum bending moment

i) Consider the pulley

The total horizontal load $= \left(T_1 + T_2 \right) \leftarrow$

Where, *the ratio of belt tensions* $= \dfrac{T_1}{T_2} = 3$

Therefore, $T_1 = 3T_2$

Also, $M_t = \left(T_1 + T_2 \right) r_p$

Where $r_p = \dfrac{600}{2} mm$

$$238750 = \left(3T_1 - T_2 \right) \times \frac{600}{2}$$

Therefore, $T_2 = 397.92N$

$T_1 = 1193.76N$

Therefore, *Total horizontal load* $= \left(T_1 + T_2 \right) = 1591.68N \leftarrow$

$$Total\ vertical\ load \quad = W_p \downarrow = 1000N \downarrow$$

ii) Consider the pinion

The total vertical load $= F_t \downarrow = \dfrac{M_t}{r_{Pinion}} \downarrow$

$$= \frac{238750}{100} = 2387.5N \downarrow$$

The total horizontal load $= F_r = F_t \tan\beta \leftarrow$

$$= 2387.5 \times \tan 20$$

$$= 868.98N \leftarrow$$

a) Horizontal load diagram

At pinion, the horizontal load $= 868.98N \leftarrow$

At pulley, the horizontal load $= (T_1 + T_2) = 1591.68N \leftarrow$

Note: The reactions at A and B are R_{AH} and R_{BH}. Since both the pinion and the pulley are overhanging, their extensions from the bearings may be considered as cantilevers with point loads at pinion and the pulley.

Figure-5.16

Note: The reactions R_{AH} and R_{BH} do not appear in the calculation of bending moment as both pinion and pulley overhang from bearings and hence they are not calculated (to save time).

b) The horizontal BMD

BM is zero at the pinion and the pulley.

BM at bearing A is $868.95 \times 200 = 173796$ N-mm

BM at bearing B is $1591.68 \times 400 = 6336672$ N-mm

c) Vertical load diagram

Vertical load at the pinion is $F_t = 2387.5N \downarrow$

Vertical load at pulley $= W_p = 1000N \downarrow$

d) Vertical BMD

BM is zero at the pinion and the pulley.

BM at bearing A is 477500 N-mm

BM at bearing B is 400000 N-mm

Resultant bending moment

At the bearing *A*,

$$M_{b1} = \sqrt{(173769)^2 + (477500)^2} = 508144.96 \text{ N-mm}$$

At the bearing *B*,

$$M_{b2} = \sqrt{(636672)^2 + (400000)^2} = 751898.42 \text{ N-mm}$$

Therefore, $M_b = M_{b2} = 751898.42 \text{ N-mm} \left(\textit{Larger of the two values}\right)$

$$D = \left[\frac{16}{\pi \times 40}\left\{(1.75 \times 751898.42)^2 + (1.25 \times 238750)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$= 57mm$$

Use shaft diameter $D = 60mm\left(\textit{Standard size}\right)$

Example 5.7: A steel shaft 750mm long supported between bearings has a cast iron pulley of 600mm diameter weighing 800N overhanging the right bearing by 200mm. This pulley receives 25kW at 1000 rpm by a belt inclined at 60^0 to the horizontal (inclined upward). The power from the shaft is transmitted through a 14.5^0 spur pinion of module 5mm having 40 teeth to a spur gear mounted directly above the pinion. The pinion is keyed to the shaft at a distance of 150mm to the right of the left bearing. Taking the ratio of belt tensions as 3:1, ultimate stress and yield stress for the material of shaft as 500MPa and 310MPa respectively, determine the shaft diameter. Use $K_b = 2$ and $K_t = 1.5$.

Solution:

The shaft is subjected to *combined bending and torsion*. Therefore, the shaft diameter is given by

$$D = \left[\frac{16}{\pi \tau_{ed}}\left\{(K_b M_b)^2 + (K_t M_t)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad \text{... E(14-12)}$$

Where, $K_b = 2.0$

$K_t = 1.5$

Torque, $M_t = \dfrac{9550N \times 10^3}{n}$ N-mm

$$= \frac{9550 \times 25 \times 10^3}{1000} = 238750 \text{ N-mm}$$

To find M_b, the maximum bending moment

i) Consider the pulley

To find T_1 *and* T_2, the belt tensions

We have, ratio of belt tensions,

$$\frac{T_1}{T_2} = 3:1(given)$$

Therefore, $T_1 = 3T_2$ ... (1)

Torque $M_t = (T_1 - T_2)r_p$

Where, $r_p = Radius\ of\ pulley = \dfrac{600}{2} mm$

Therefore, $238750 = (3T_1 - T_2)\dfrac{600}{2}$

$T_2 = 795.83N$

$T_1 = 2387.49N$

Figure-5.17

Figure-5.18

Therefore, *Horizontal load on Pulley* $= (2387.49 + 795.83)\cos 60$

$= 1591.66N$

Vertical load on Pulley

$= \text{Difference of } (2387.49 + 795.83)\sin 60 \uparrow \text{ and } 800 \downarrow$

$= \text{Difference of } 2756.85 \uparrow \text{ and } 800 \downarrow = 1956.85N \uparrow$

ii) Consider the Pinion

Total horizontal load $= F_t = \frac{M_t}{r_{Pinion}}$

Here, $r_{Pinion} = \frac{d_{Pinion}}{2} = \frac{mZ_1}{2}$

Where $m = Module = 5mm$

$Z_1 = Number\ of\ teeth\ on\ Pinion = 40$

$$r_{Pinion} = \frac{5 \times 40}{2} = 100mm$$

Therefore, $F_t = \frac{238750}{100} = 2387.5N$

Vertical load on the Pinion $= F_r = F_t \tan\beta \downarrow$

$$= 2387.5 \tan 14\tfrac{1}{2} \downarrow = 617.5N \downarrow$$

Load and Bending Moment Diagrams

a) Horizontal load diagram

On the pulley, the horizontal load $= 1591.66N$

On the pinion, the horizontal load $= F_t = 2387.5N$

Let R_{AH} and R_{BH} be the reactions at A and B due to horizontal loads only.

$$R_{AH} + F_t + R_{BH} = 1591.66 \qquad \text{... (1)}$$

Therefore, $R_{AH} + R_{BH} = 1591.66 - F_t$

$$= 1591.66 - 2387.5 = -795.84N$$

Taking moment about A

$$1591.66 \times 950 = F_t \times 150 + R_{BH} \times 750$$

$$= 2387.5 \times 150 + R_{BH} \times 750$$

Therefore, $R_{BH} = \frac{1591.66 \times 950 - 2387.5 \times 150}{750} = 1538.6N$

$$R_{AH} = -795.84 - 1538.6 = -2334.44N$$

b) Horizontal BM diagram

Bending moment is zero at bearing A and pulley. BM is 350166 N-mm at the Pinion.

Bending moment is 318332 N-mm at bearing B.

c) Vertical load diagram

Vertical load on Pinion is $F_r = 617.5N \downarrow$

Vertical load on Pulley $= 1956.85N \uparrow$

Let R_{AV} and R_{BV} be the reactions at *A* and *B* due to vertical loads only.

Then $R_{AV} + R_{BV} + 1956.85 = 617.5$

$$(Total\ upward\ load) = (Total\ downward\ load)$$

Therefore, $R_{AV} + R_{BV} = -1339.34N$

Taking moments about *A*,

$$1956.85 \times 950 + R_{BV} \times 750 = 617.5 \times 150$$

Therefore, $R_{BV} = \dfrac{617.5 \times 150 - 1956.85 \times 950}{750}$

$$= \frac{92625 - 1859007.5}{750}$$

$$= -2355.18N$$

Therefore, $R_{AV} = -1339.35 - (-2355.18)$

$$= 1015.83N$$

d) Vertical BM diagram

Bending moment is zero at bearing *A* and pulley.

Bending moment at Pinion is 152374.5 N-mm

Bending moment at bearing *B* is 393170 N-mm

Resultant Bending moment at Pinion

$$M_{b1} = \sqrt{(350166)^2 + (152374.5)^2} = 381882.46 \text{ N-mm}$$

Resultant Bending moment at Bearing B,

$$M_{b2} = \sqrt{(318332)^2 + (391370)^2} = 504485.6 \text{ N-mm}$$

Therefore, $M_b = M_{b2} = 504485.62 \text{ N-mm} (larger\ of\ the\ above\ two\ values)$

For the material of shaft,

$$\sigma_u = 500MPa$$

$$18\% \ of \ \sigma_u = 0.18 \times 500$$
$$= 90MPa$$
$$\sigma_y = 310MPa$$

Therefore, $30\% \ of \ \sigma_y = 0.3 \times 310$
$$= 93MPa$$

Note: According to ASME code,

$$\tau_{ed} = 75\% \ of \ the \ smaller \ of \ the \ above \ two \ values$$
$$= 75\% \ of \ 90MPa = 67.5MPa$$

$$D = \left[\frac{16}{\pi \times 67.5} \left\{ \left(2.00 \times 504485.62\right)^2 + \left(1.5 \times 238750\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$
$$= 43.22mm$$

Use, shaft diameter $D = 45mm\left(Standard \ size\right)$

Example 5.8: A hollow shaft mounted between bearings 9.5m apart transmits 10000kW at 90rpm. The shaft weighs 66220N. The outside and inside diameters are 450mm and 300mm respectively. Determine

a) The shear stress induced in the shaft assuming that the weight of the shaft acts as its mid length.

b) The angular deflection between bearings.

Solution:

The shaft is subjected to *combined bending and torsion* and hence the outside diameter of the hollow shaft is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-37)}$$

Where, $k = \frac{D_i}{D_0}$

$$= \frac{300}{450} = \frac{2}{3} = 0.67$$

$K_b = 1.5$

$K_t = 1.0$ (Assuming steady load) ... T(14-2)

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 10000 \times 10^3}{90} \text{ N-mm}$$

$$= 1.061 \times 10^9 \text{ N-mm}$$

To find M_b, the maximum bending moment

The width of the shaft acts as its mid point and the reactions at the bearings are equal and are given by

$$\text{Reaction} = R_A = R_B = \frac{66220}{2} = 33110 N$$

Then the bending moment

$$M_b = Reaction \times \frac{L}{2}$$

Where, $L = Length\ of\ shaft$

$$M_b = 33110 \left(\frac{9.5 \times 10^3}{2} \right) \text{ N-mm}$$

$$= 1.5727 \times 10^8 \text{ N-mm}$$

Substituting these values we get,

$$450 = \left[\frac{16}{\pi \tau_{ed} \left(1 - 067^4\right)} \left\{ \left(1.57 \times 1.5727 \times 10^8\right)^2 + \left(1.00 \times 1.061 \times 10^9\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Therefore, $\tau_{ed} = 76.08\ N/mm^2$

Therefore, the shear stress induced in the shaft $\tau_{ed} = 76.08\ N/mm^2$

b) To find angular deflection θ,

For hollow shaft,

$$D_0 = \left[\frac{584 M_t L}{G\theta\left(1-k^4\right)}\right]^{\frac{1}{4}} \qquad \text{... E(14-42)}$$

$$450 = \left[\frac{584 \times 1.061 \times 10^9 \times 9.5 \times 10^3}{80 \times 10^3 \times \theta\left(1-0.67^4\right)}\right]^{\frac{1}{4}}$$

Therefore, $\theta = 2.247^0$

Example 5.9: Prove that a hollow shaft is stronger and stiffer than a solid shaft of same length and weight.

Solution:

Let *'d'* be the diameter of solid shaft and $'d_0'$ and $'d_i'$ be the outside and inside diameters of hollow shaft.

a) To prove that hollow shaft is stronger than solid shaft.

Torque transmitted by solid shaft is

$$M_{ts} = \frac{\pi d^3}{16}\tau_s \qquad \text{... (i)}$$

Similarly, torque transmitted by hollow shaft is

$$M_{th} = \frac{\pi}{16}\frac{\left(d_0^4 - d_i^4\right)}{d_0}\tau_h \qquad \text{...(ii)}$$

Therefore,

$$\frac{M_{th}}{M_{ts}} = \frac{\frac{\pi}{16}\frac{\left(d_0^4 - d_i^4\right)}{d_0}\tau_h}{\frac{\pi d^3}{16}\tau_s}$$

$$= \frac{d_0^4 - d_i^4}{d^3 d_0} \qquad \left(Since \;\; \tau_h = \tau_s\right) \qquad \text{... (iii)}$$

Considering weight of shafts,

Weight of solid shaft = Weight of hollow shaft (given)

$$\frac{\pi d^2}{4} l\rho = \frac{\pi}{4}\left(d_0^2 - d_i^2\right) l\rho$$

Where, $l = Length\ of\ shafts$

$\rho = Density\ of\ materail\ of\ shaft$

Therefore, $d^2 = d_0^2 - d_i^2$ ... (iv)

From equation (iii)

$$\frac{M_{th}}{M_{ts}} = \frac{\left(d_0^2 + d_i^2\right)\left(d_0^2 - d_i^2\right)}{d^3 d_0} \quad \text{... (v)}$$

From equation (iv)

$$d^3 = d\left(d_0^2 - d_i^2\right)$$

Substituting $d^3 = d\left(d_0^2 - d_i^2\right)$ in equation (v), we get

$$\frac{M_{th}}{M_{ts}} = \frac{\left(d_0^2 + d_i^2\right)\left(d_0^2 - d_i^2\right)}{d\left(d_0^2 - d_i^2\right)d_0}$$

$$= \frac{\left(d_0^2 + d_i^2\right)}{dd_0}$$

Dividing the denominator and numerator of the above equation by d_0^2

$$\frac{M_{th}}{M_{ts}} = \frac{\dfrac{d_0^2 + d_i^2}{d_0^2}}{\dfrac{dd_0}{d_0^2}}$$

$$= \frac{1 + \dfrac{d_i^2}{d_0^2}}{\dfrac{d}{d_0}}$$

$$= \frac{\dfrac{d_0^2 + d_i^2}{d_0^2}}{\dfrac{\sqrt{d_0^2 - d_i^2}}{d_0}}$$

Since $d^2 = d_0^2 - d_i^2 \quad and \quad d = \sqrt{d_0^2 - d_i^2}$

Therefore, $$\frac{M_{th}}{M_{ts}} = \frac{\dfrac{d_0^2 + d_i^2}{d_0^2}}{\sqrt{\dfrac{d_0^2 - d_i^2}{d_0}}}$$

$$= \frac{1 + \left(\dfrac{d_i}{d_0}\right)^2}{\sqrt{1 - \left(\dfrac{d_i}{d_0}\right)^2}}$$

Since $\dfrac{d_i}{d_0} < 1,$

$$1 + \left(\frac{d_i}{d_0}\right)^2 > \sqrt{1 - \left(\frac{d_i}{d_0}\right)^2}$$

Therefore, $$\frac{M_{th}}{M_{ts}} > 1$$

$$M_{th} > M_{ts}$$

i.e., the torque transmitted by hollow shaft is greater than the torque transmitted by solid shaft or the *Hollow shaft is stronger than a solid shaft.*

b) To prove that the hollow shaft is stiffer than solid shaft

We have, $$\frac{M_t}{J} = \frac{G\theta}{l}$$

Stiffness $\frac{M_t}{\theta} = \frac{JG}{l}$

For hollow shaft,

$$J_h = \frac{\pi\left(d_0^4 - d_i^4\right)}{32}$$

For solid shaft,

$$J_s = \frac{\pi d^4}{32}$$

If θ_h and θ_s are the angle of twist of hollow and solid shafts respectively, then

$$\frac{M_{th}}{\theta_h} = \frac{\pi\left(d_0^4 - d_i^4\right)G_h}{32l}$$

$$\frac{M_{ts}}{\theta_s} = \frac{\pi\left(d^4\right)G_s}{32l}$$

Since same material, $G_s = G_h$

$$\frac{\frac{M_{th}}{\theta_h}}{\frac{M_{ts}}{\theta_s}} = \frac{\frac{\pi\left(d_0^4 - d_i^4\right)}{32l}}{\frac{\pi d^4}{32l}}$$

$$= \frac{d_0^4 - d_i^4}{d^4}$$

$$= \frac{\left(d_0^2 + d_i^2\right)\left(d_0^2 - d_i^2\right)}{d^2 \times d}$$

But $d^2 = d_0^2 - d_i^2\left(Proved\right)$

Therefore, the ratio becomes

$$= \frac{\left(d_0^2 + d_i^2\right)\left(d_0^2 - d_i^2\right)}{\left(d_0^2 - d_i^2\right)\left(d_0^2 - d_i^2\right)}$$

$$= \frac{\left(d_0^2 + d_i^2\right)}{\left(d_0^2 - d_i^2\right)}$$

Dividing the Numerator and Denominator by d_0^2

$$\frac{\dfrac{M_{th}}{\theta_h}}{\dfrac{M_{ts}}{\theta_s}} = \frac{\left(\dfrac{d_0^2 + d_i^2}{d_0^2}\right)}{\left(\dfrac{d_0^2 - d_i^2}{d_0^2}\right)}$$

$$= \frac{1 + \left(\dfrac{d_i}{d_0}\right)^2}{1 - \left(\dfrac{d_i}{d_0}\right)^2}$$

$$\frac{d_i}{d_0} < 1$$

Therefore, $$1 + \left(\frac{d_i}{d_0}\right)^2 > 1 - \left(\frac{d_i}{d_0}\right)^2$$

i.e., $\dfrac{\dfrac{M_{th}}{\theta_h}}{\dfrac{M_{ts}}{\theta_s}} > 1$ i.e., a hollow shaft is stiffer than a solid shaft of same weight and length.

Example 5.10: A solid shaft and a hollow shaft are to have equal strength in torsion. The hollow shaft is the be 10% larger in diameter than the solid shaft. Determine the ration of weight of hollow shaft to that of the solid shaft. Both the shafts are made of same material.

Solution:

Let *'d'* be the diameter of solid shaft and $'d_0'$ and $'d_i'$ be the dimensions of the hollow shaft transmitting torque M_t

Then $$M_{ts} = \frac{\pi d^3}{16}\tau_s \qquad \text{... (1)}$$

$$M_{th} = \frac{\pi\left(d_0^4 - d_i^4\right)}{16 d_0}\tau_h \qquad \text{... (2)}$$

From data, for equal strength in torsion $M_{ts} = M_{th}$

i.e., $$\frac{\pi d^3}{16}\tau_s = \frac{\pi}{16}\frac{\left(d_0^4 - d_i^4\right)\tau_h}{d_0}$$

$$d^3 = \frac{\left(d_0^4 - d_i^4\right)}{d_0}$$

Since $\tau_s = \tau_h$ materials of shafts are same.

Here $$d_0 = 1.1d\left(data\right)$$

Therefore $$d^3 = \left[\frac{\left(1.1d\right)^4 - d_i^4}{1.1d}\right]$$

$$1.1d^4 = \left(1.1d\right)^4 - d_i^4$$

$$1.1d^4 = 1.4641d^4 - d_i^4$$

$$\frac{d_i^4}{d^4} = 0.3641$$

Therefore $$\frac{d_i}{d} = 0.7768$$

Since $d = \frac{d_0}{1.1}$

$$\frac{d_i}{\frac{d_0}{1.1}} = 0.7768$$

$$\frac{d_i}{d_0} = 0.7062 = k$$

Therefore, $\dfrac{\dfrac{Weight\ of\ hollow\ shaft}{Unit\ length}}{\dfrac{Weight\ of\ solid\ shaft}{Unit\ length}} = \dfrac{\dfrac{\pi}{4}\left(d_0^2 - d_i^2\right) l\rho}{\dfrac{\pi}{4} d^2 \times 1\rho}$

Where $\rho = Density\ of\ the\ material\ of\ shaft$

Therefore $\dfrac{W_H}{W_s} = \dfrac{d_0^2 - d_i^2}{d^2}$

$$= \frac{d_0^2 - \left(0.7062 d_0\right)^2}{\left(\dfrac{d_0}{1.1}\right)^2}$$

$$= 0.50128 \times \left(1.1\right)^2$$

$$= 0.6065$$

i.e., $W_H = 0.6065 W_s$

Therefore, Wight of hollow shaft $= 0.6065 \times Weight\ of\ solid\ shaft$

Example 5.11: A commercial steel shaft 1m long supported in a square bearing carries a cast iron pulley of diameter 500mm weighing 600N at its other end. The pulley receives 25kW at 1000 rpm from an electric motor mounted directly below it by a flat belt drive. Taking angle of lap of belt on pulley as 180^0, and coefficient of friction 0.3 determine,

a) The diameter of solid shaft required.
b) The bearing pressure at the bearing.

Solution:
The shaft is subjected to *combined bending and torsion*.
Therefore, the shaft diameter is obtained from equation

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{\left(K_b M_b\right)^2 + \left(K_t M_t\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $\tau_{ed} = 40\ N/mm^2$, for commercial steel as per ASME code.

$K_b = 1.5$

$K_t = 1.0$ ... T(14-2)

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 25 \times 10^3}{1000} = 238750 \text{ N-mm}$$

To find M_b, the maximum bending moment

The vertical load on Pulley $= \left(T_1 + T_2 + W_p\right) \downarrow$

Where $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.03 \times 180 \times \frac{\pi}{180}} = 2.57$

Therefore, $T_1 = 2.57 T_2$

Also, $M_t = \left(T_1 - T_2\right) r_{Pulley}$

$$238750 = \left(2.5 T_2 - T_2\right) \frac{500}{2}$$

Therefore, $T_2 = 636.7 N$

$T_1 = 1636.32 N$

$W_p = Weight\ of\ \ pulley = 600 N \downarrow$

Therefore, Vertical load on pulley

$$= \left(T_1 + T_2 + W_p\right) \downarrow$$

$$= \left(1636.32 + 636.7 + 600\right) N \downarrow$$

$$= 2873.02 N \downarrow$$

Figure-5.19

Where $M_b = Vertical\ load\ on\ pulley \times 1000$

$= 2873.02 \times 1000$ N-mm

2873020 N-mm

Shaft diameter $D = \left[\frac{16}{\pi \times 40} \left\{ \left(1.5 \times 2873020\right)^2 + \left(1 \times 238750\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$

$= 81.91mm$

Use shaft diameter $D = 85mm \ (Standard\ size)$

b) To find the bearing pressure P_b at the bearing

Bearing pressure

$$P_b = \frac{Bearing\ load}{Projected\ area\ of\ bearing}$$

Here, Bearing load $= (T_1 + T_2 + W_p)$

$= 2873.02N \downarrow$

Projected area of bearing

$= LD$

Where L is the length of bearing

For a square bearing,

$L = D = 85mm$

Therefore, Bearing pressure

$$P_b = \frac{2873.02}{85 \times 85} = 0.3977\ N/mm^2$$

Example 5.12: A steel shaft 600mm long supported between bearings carries a pulley of diameter 400mm, weighing 400N and is mounted at the middle of shaft. This shaft receives 40kW at 600 rpm by a flat belt drive. Power from the shaft is transmitted through another pulley of diameter 600mm weighing 600N overhanging the right bearing by 200mm. The belt drives on the pulley are at right angles to each other. Taking the ratio of belt tensions as 3, determine the diameter of the shaft required taking commercial steel as the shaft material.

Solution:

The shaft is subjected to *combined bending and torsion* and hence the shaft diameter obtained from equation,

$$D = \left[\frac{16}{\pi\tau_{ed}}\left\{(K_b M_b)^2 + (K_t M_t)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad \text{... E(14-12)}$$

Where $\tau_{ed} = 40\ N/mm^2$ (for commercial steel as per ASME code)

$K_b = 1.5$

$K_t = 1$ ... T(14-2)

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 40 \times 10^3}{600} = 636666.67 \text{ N-mm}$$

To find $M_{b,}$ the maximum bending moment

Consider the pulley mounted between the bearings. Assuming the belt drive vertical, let T_1 and T_2 be the belt tensions.

Therefore, Total vertical load on pulley 1 is $= \left(T_1 + T_2 + W_p\right) \downarrow$

To find T_1 *and* T_2

Given, ratio of belt tensions $= \frac{T_1}{T_2} = 3$

Therefore, $T_1 = 3T_2$ ... (1)

Also, $M_t = \left(T_1 - T_2\right) r_{p_1}$

Where $r_{p_1} = Radius\ of\ \ pulley = \frac{400}{2} mm$

Therefore, $636666.67 = \left(3T_2 - T_1\right)\frac{400}{2}$

$T_2 = 1591.67 N$

$T_1 = 4775 N$

$W_{p_1} = Weight\ of\ \ pulley\, 1 = 400 N \downarrow$

Vertical load on pulley $= \left(4775 + 1591.67 + 400\right) = 6766.67 N \downarrow$

Horizontal load on pulley = 0

Consider pulley 2

Let T_1' and T_2' be the tensions acting normal to T_1 and T_2

To find T_1' *and* T_2'

Given $\frac{T_1'}{T_2'} = 3$

Therefore, $T_1' = 3T_2'$

Also $M_t = \left(T_1' = T_2'\right) r_{p_2}$

Where $r_{p_2} = \frac{600}{2} mm$

i.e., $636666.67 = \left(3r_2' - T_2'\right)\frac{600}{2}$

$T_2' = 1061.11N$

And $T_1' = 3183.33N$

Therefore, Horizontal load $= \left(T_1' + T_2'\right) = 4244.44N \rightarrow$

Vertical load $= W_{p_2} = 600N \downarrow$

To draw load and BM diagrams

a) Horizontal load diagram

Let R_{AH} and R_{BH} be the reactions at *A* and *B* due to horizontal load. Then,

$$R_{AH} + R_{BH} = 4244.44N$$

Taking moments about *A*,

$$R_{BH} \times 600 = 4244.44 \times 800$$

Therefore, $R_{BH} = 5659.25N$

And $R_{AH} = -1414.81N$

b) Horizontal BM diagram

BM is zero at bearing *A* and pulley 2

BM at pulley 1 is $1414.81 \times 300 = 24443$ N-mm

BM at bearing *B* is $4244.44 \times 200 = 848888$ N-mm

Figure-5.20

c) Vertical load diagram

Let R_{AV} and R_{BV} be the reactions at *A* and *B* due to vertical loads only

$$R_{AV} + R_{BV} = 6766.67 + 600$$
$$= 7366.67N$$

Taking moment about *A*,

$$R_{BV} \times 600 = 600 \times 800 + 6766.67 \times 300$$

Therefore, $R_{BV} = 4183.34N \uparrow$

And $R_{AV} = 3183.33N \uparrow$

d) Vertical BM diagram

BM is zero at bearing *A* and pulley 2.

BM is 954999 N-mm at pulley 1 and 120000 N-mm at bearing *B*.

Resultant M_b (Maximum bending moment)

Resultant BM at pulley 1 is

$$M_{b1} = \sqrt{(424443)^2 + (954999)^2} = 1045071.7 \text{ N-mm}$$

Resultant BM at bearing *B* is

$$M_{b2} = \sqrt{(848888)^2 + (120000)^2} = 857327.73 \text{ N-mm}$$

Therefore, Maximum bending moment $M_b = M_{b1} = 1045071.7$ N-mm (Larger of the two values of M_b)

Therefore, $$D = \left[\frac{16}{\pi \times 40} \left\{ (1.5 \times 1045071.7)^2 + (1 \times 636666.7)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Use shaft diameter $D = 60mm$

Example 5.13: A steel shaft 1 meter long supported between bearings has a spur pinion of module 5mm having 40 teeth keyed to the shaft at a distance of 200mm to the right of the left bearing and receives 20kW at 1000 rpm from another gear mounted on a parallel shaft below it. Power from the shaft is transmitted through another gear of module 8mm having 50 teeth, keyed to the shaft at a distance of 100mm to the left of the right bearing to a second gear mounted behind it. Taking the pressure angle of gears 14½^0, neglecting the weights of gears, determine the diameter of the solid shaft required. Use shock and fatigue factors K_b and K_t as 2 and 1.5 respectively. The material of the shaft has an ultimate strength of $500\ N/mm^2$ and yield point of $30\ N/mm^2$.

Solution:

The shaft is subjected to *combined bending and torsion*

Therefore *D*, the diameter of shaft subjected to combined bending and torsion is obtained from equation

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where $K_b = 2.0$

$K_t = 1.5$ ... T(14-2)

For the material of shaft,

$$\sigma_u = 500\ N/mm^2;\ 18\%\ of\ \sigma_u = \frac{18}{100} \times 500 = 90 MPa$$

$$\sigma_y = 310\ N/mm^2;\ 30\%\ of\ \sigma_y = \frac{30}{100} \times 310 = 93 MPa$$

According to ASME code

$\tau_{ed} = 75\%$ of the smaller of the two values of $\left(\sigma_u\ and\ \sigma_y\right)$ when key is used.

$$= 75\%\ of\ 90 MPa$$

$$= 0.75 \times 90 = 67.5 MPa$$

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 20 \times 10^3}{1000} \text{ N-mm}$$

$$= 191000 \text{ N-mm}$$

To find M_b, the maximum bending moment

Consider the pinion mounted on shaft

The horizontal load on pinion,

$$F_{t_1} = \frac{M_t}{r_1}$$

Where $r_1 = \frac{d_1}{2} = \frac{m_1 Z_1}{2}$

$$m_1 = Module;\ Number\ of\ teeth = 40$$

$$r_1 = \frac{5 \times 40}{2} = 100mm$$

$$F_{t_1} = \frac{191000}{100} = 1910N \rightarrow$$

The vertical load on pinion,

$$= F_{r_1} = F_{t_1} \tan\beta \uparrow$$

$$= 1910 \tan 14\tfrac{1}{2} \uparrow$$

$$= 493.96N \uparrow$$

Consider the gear mounted on the shaft

$$Vertical\ load\ on\ gear = F_{t_2} \uparrow = \frac{M_t}{r_2} \uparrow$$

Where $r_2 = \frac{d_2}{2} = \frac{m_2 Z_2}{2}$

$$= \frac{8 \times 50}{2} = 200mm$$

Therefore, $F_{t_2} = Vertical\ load = \frac{191000}{200} = 955N \uparrow$

$$F_{r_2} = Horizontal\ load = F_{t_2} \tan\beta$$

$$= 955 \tan 14\tfrac{1}{2} = 246.98N$$

To draw load and BM diagrams

a) Horizontal load diagram

$$R_{AH} + R_{BH} + 246.98 = 1910N$$

Therefore, $R_{AH} + R_{BH} = 1663.02N$ ... (i)

Taking moments about *A*,

$$R_{BH} \times 1000 + 246.98 \times 900 = 1910 \times 200$$

Therefore, $R_{BH} = 159.72N$

$$R_{AH} = 1663.02 - R_{BH}$$

$$= 1663.02 - 159.72 = 1503.3N$$

Figure-5.21

b) Horizontal BMD

BM is zero at bearings *A* and *B*

At pinion, bending moment

$$= R_{AH} \times 200$$

$$= 1503.3 \times 200 = 300660 \text{ N-mm}$$

At gear, bending moment

$$= R_{BH} \times 100$$

$$= 159.72 \times 100 = 15972 \text{ N-mm}$$

c) Vertical load diagram

Vertical load on pinion $= 493.96N \uparrow$

Vertical load on gear $= 955N \uparrow$

Let R_{AV} and R_{BV} be the reactions due to vertical loads. Then

$$R_{AV} + R_{BV} = 955 + 493.96$$

$$= 1448.96N$$

Taking moments about *A*,

$$R_{BV} \times 1000 = 955 \times 900 + 493.96 \times 200$$

$$R_{BV} = 958.29N$$

And $R_{AV} = 490.67N$

d) Vertical BMD

BM is zero at bearings *A* and *B*.

At pinion, the bending moment

$$= R_{AV} \times 200$$

$$= 490.67 \times 200 = 98134 \text{ N-mm}$$

(Note: It is shown acting downwards because R_{AV} is negative)

At gear, the bending moment

$$= R_{BV} \times 100$$
$$= 958.29 \times 100 = 95829 \text{ N-mm}$$

To find M_b, the maximum bending moment on the shaft

Resultant bending moment at pinion is

$$M_{b_1} = \sqrt{(300660)^2 + (98134)^2}$$
$$= 316270 \text{ N-mm}$$

Resultant bending moment at gear is

$$M_{b_2} = \sqrt{(15972)^2 + (95829)^2}$$
$$= 97150.92 \text{ N-mm}$$

Therefore, Maximum bending moment occurs at pinion and hence $M_b = M_{b1} = 316270$ N-mm

Therefore, $$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ (K_b M_b)^2 + (K_t M_t)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= \left[\frac{16}{\pi \times 67.5} \left\{ (2 \times 316270)^2 + (1.5 \times 191000)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 37.4 mm$$

Use shaft diameter $D = 40mm\,(Standard\ size)$

Example 5.14: A 45C8 (C45) shaft transmitting 15kW at 210 rpm is supported between two bearings 1000mm apart has two gears keyed to it. The pinion having 24 teeth and module 6mm is located 200mm to the right of the left bearing and delivers power to a gear mounted behind it. The gear having 80 teeth of module 6mm is located 100mm to the left of the right bearing and receives power from a pinion such that the tangential force acts horizontal. Taking combined shock and fatigue factors 1.75 in bending and 1.25 in torsion, determine the shaft diameter. (VTU-Dec 08/Jan 09)

Solution:

The shaft is subjected to *combined bending and torsion.*

Therefore, the shaft diameter *D* is obtained from

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \quad \text{... E(14-12)}$$

Where $K_b = 1.75$

$K_t = 1.25(given)$

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 15 \times 10^3}{210} = 682142.86 \text{ N-mm}$$

Figure-5.22

For 45C8, $\sigma_{ut} = 650 \, N/mm^2$ ... T(1-25)

Assuming $\sigma_{yt} = 0.6 \sigma_{ut}$

$$= 0.6 \times 650 = 390 \, N/mm^2$$

$$\tau_y = \frac{\sigma_{yt}}{2} = 195 \; N/mm^2$$

$$\tau_{ed} = \frac{\tau_y}{factor \; of \; safety}$$

$$= \frac{195}{3} = 65.33 \, N/mm^2$$

To find M_b, the maximum bending moment

Consider the pinion 1

The vertical load

$$F_{t_1} = \frac{M_{t_1}}{r_{Pinion}} \uparrow$$

Where $r_{Pinion} = \frac{d_{Pinion}}{2}$

$$= \frac{m_1 Z_1}{2} = \frac{6 \times 24}{2} = 72mm$$

Therefore, $F_{t_1} = Vertical\ load = \frac{682142.86}{72} = 9474.21N \uparrow$

$$F_{r_1} = Horizontal\ load = F_{t_1} \tan\beta$$

$$= 9474.21 \tan 14.5$$

$= 2450.2N$ (Assuming pressure angle $\beta = 14.5^0$)

Consider the gear 2,

$$F_{t_2} = Horizontal\ load = \frac{M_{t_2}}{r_{Gear}} \rightarrow$$

$$r_{Gear} = \frac{d_{Gear}}{2}$$

$$= \frac{m_2 Z_2}{2} = \frac{6 \times 80}{2} = 240mm$$

Therefore, $F_{t_2} = \frac{682142.86}{240} = 2842.26N \rightarrow$

$$F_{r_2} = Vertical\ load\ on\ gear = F_{r_2} = F_{t_2} \tan\beta \uparrow$$

$$= 2842.26 \tan 14.5$$

$$= 735.06N \uparrow$$

a) Horizontal load diagram

Horizontal load on gear $= F_{t_2} = 2842.26N \rightarrow$

Horizontal load on pinion $= F_{r_1} = 2450.2N \leftarrow$

Let R_{AH} and R_{BH} be the reactions at bearings A and B due to horizontal load only. Then,

$$R_{AH} + R_{BH} = 2842.26 + 2450.2$$

$$= 5292.46N$$

Taking moments about *A*,

$$R_{BH} \times 750 = 2450.2 \times 650 + 2842.26 \times 150$$

Therefore, $R_{BH} = 2691.96N$

$$R_{AH} = 2600.5N$$

b) Horizontal BM diagram

BM is zero at bearings *A* and *B*

At gear, $BM = 2600.5 \times 150$

$= 390075$ N-mm

At pinion, $BM = 2691.96 \times 100$

$= 269196$ N-mm

c) Vertical load diagram

Let R_{AV} and R_{BV} be the reactions at *A* and *B* due to vertical loads only

Then $R_{AV} + R_{BV} = 9474.21 + 735.06$

$= 10209.26$

Also taking moments about *A*,

$$R_{BV} \times 750 = 7474.21 \times 650 + 735.06 \times 150$$

Therefore, $R_{BV} = 8357.99N$

$$R_{AV} = 1851.27N$$

d) Vertical BM diagram

BM is zero at bearings *A* and *B*.

At gear, $BM = 675.18 \times 150$

$= 101277$ N-mm

At pinion, $BM = 8063.97 \times 100$

$= 806397$ N-mm

Resultant bending moment

At gear, resultant bending moment

$$M_{b_1} = \sqrt{(390075)^2 + (277690)^2}$$

$$= 478822.72 \text{ N-mm}$$

At pinion, resultant bending moment

$$M_{b_2} = \sqrt{(269196)^2 + (835799)^2}$$

$$= 878081.12 \text{ N-mm}$$

Therefore, Maximum bending moment occurs at the pinion.

Therefore, $M_b = M_{b_2} = 878081.12 \text{ N-mm}$

$$D = \left[\frac{16}{\pi \times 65} \left\{ (1.75 \times 878081.12)^2 + (1.25 \times 682142.86)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 52.088 mm$$

Use shaft diameter $D = 55mm\left(Standard\ size\right)$

Example 5.15: Determine the diameter of a hollow shaft with diameter ratio of 0.6 to transmit 40kW at 750 rpm and also to sustain a bending moment of 60 Nm limiting the angle of twist to 0.5^0 in a length of 1000mm. Select suitable material.

Solution:

The hollow shaft is subjected to *combined bending and torsion.*

Therefore, the outside diameter D_0 based on *torque capacity* is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} \left\{ (K_b M_b)^2 + (K_t M_t)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-37)}$$

Where $k = \frac{D_i}{D_0} = 0.6$

$$K_b = 1.5$$

$$K_t = 1\left(Assuming\ steady\ load\right) \qquad \text{... T(14-2)}$$

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 40 \times 10^3}{750} = 5305555.6 \text{ N-mm}$$

Select 45C8 as material of shaft,

$$\sigma_{ut} = 618 - 696\, N/mm^2 \text{, } say\ \sigma_{ut} = 650\, N/mm^2 \qquad \text{... T(1-25)}$$

$$\sigma_{yt} = 0.6\sigma_{ut}$$

$$= 0.6 \times 650 = 390 MPa$$

$$\tau_y = \frac{\sigma_{yt}}{2}$$

$$= \frac{390}{2} = 195 MPa$$

Therefore, $$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{195}{3} = 65\, N/mm^2$$

Therefore,

$$D_0 = \left[\frac{16}{\pi \times 65\left(1 - 0.6^4\right)} \left\{ \left(1.5 \times 600 \times 10^3\right)^2 + \left(1.25 \times 5305555.6\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Therefore, $$D_0 = 84.46 mm$$

$$D_i = 0.6 D_0$$

$$= 0.6 \times 84.46 = 50.68 mm$$

b) Based on torsional rigidity, the outside diameter is given by

$$D_0 = \left[\frac{584 M_t L}{G\theta\left(1 - k^4\right)} \right]^{\frac{1}{4}} \qquad \text{... E(14-42)}$$

$$= \left[\frac{584 \times 5305555.6 \times 1000}{80 \times 10^3 \times 0.5\left(1 - 0.6^4\right)} \right]^{\frac{1}{4}}$$

$$D_0 = 97.13 mm$$

Therefore, $$D_i = 0.6 D_0$$

$$0.6 \times 97.13 = 58.28mm$$

Therefore, *Adopt outside diameter* $D_0 = 97.13mm$

and inside diameter $D_i = 58.28mm$

Example 5.16: A horizontal piece of commercial shaft is supported by two bearings 1.5m apart. A gear of 20^0 involute profile and 175mm pcd is located 400mm to the left of the right bearing and is drive by a pinion directly behind it. A 600mm diameter pulley is keyed to the shaft 600mm to the right of the bearing and drives a horizontal belt directly behind it. The ration of belt tensions is 3:1 with the slack side at the top. The driver transmits 80kW at 400 rpm. Take $K_b = 1.5$, $K_t = 1.25$. The weight of pulley is 800N.

a) Draw the bending moment diagrams.

b) Calculate the shaft diameter based on maximum shear stress theory.

Solution:

The shaft is subjected to combined *bending and torsion.* Therefore, the diameter of shaft based on *torque capacity* is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $\tau_{ed} = 40MPa\left(For\ commercial\ shaft\ according\ to\ ASME\ code\right)$

$K_b = 1.5$

$K_t = 1.25\left(given\right)$

$$M_t = \frac{9550N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 80 \times 10^3}{400} = 1910000 \text{ N-mm}$$

a) To find M_b, the maximum bending moment

i) Consider the gear keyed to the shaft

Note: Since the slack side is at the top, the pulley and hence the gear rotate in the clockwise direction.

Vertical load on Gear

$$F_t = \frac{M_t}{r_{Gear}} \uparrow$$

$$= \frac{1910000}{175} = 10914.29N \uparrow$$

Horizontal load

$$F_r = F_t \tan\beta$$

$$= 10914.29 \times \tan 20^0$$

$$= 3972.48N \leftarrow$$

ii) Consider the pulley

Vertical load on pulley $= W_p = 800N \downarrow$

Horizontal load $= (T_1 + T_2) \rightarrow$

Where $\frac{T_1}{T_2} = 3$

Therefore, $T_1 = 3T_2$

Also $M_t = (T_1 - T_2) r_{Pulley}$

$$1910000 = (3T_2 - T_2)\frac{600}{2}$$

Therefore, $T_2 = 3183.33N$

$$T_1 = 9549.99N$$

Therefore, horizontal load on pulley

$$= T_1 + T_2 = 12733.33N \rightarrow$$

Figure-5.23

To draw load and BM diagram

$$R_{AH} + R_{BH} + 3972.48 = 12733.33$$

$$R_{AH} + R_{BH} = 8760.85N$$

Taking moments about A,

$$R_{BH} \times 1500 + 3972.48 \times 1100 = 12733.33 \times 600$$

Therefore, $R_{BH} = 2180.18N$

$R_{AH} = 6580.67N$

b) Horizontal BM diagram

BM is zero at bearings A and B

At pulley, bending moment

$$= 6580.67 \times 600 = 3948402 \text{ N-mm}$$

At gear, bending moment

$$= 2180.18 \times 400 = 872072 \text{ N-mm}$$

c) Vertical load diagram

At pulley, the vertical load $= W_p = 800N \downarrow$

Let R_{AV} and R_{BV} be the reactions due to vertical loads only, then

$$R_{AV} + R_{BV} + 10914.29 = 800$$

Therefore, $R_{AV} + R_{BV} = -10114.29N$

Taking moments about A,

$$R_{BV} \times 1500 + 10914.29 \times 1100 = 800 \times 600$$

$$R_{BV} = -7683.81N$$

$$R_{AV} = -2430.49N$$

$$M_b \ at \ pulley = -17798.1 \times 600$$

$$= -10678860 \text{ N-mm}$$

$$M_b \ at \ Gear = 7683.81 \times 400$$

$$= 3073524 \text{ N-mm}$$

Resultant bending moment at pulley

$$M_{b1} = \sqrt{(3948402)^2 + (1458294)^2}$$

$$= 4209097.26 \text{ N-mm}$$

Resultant bending moment at gear

$$M_{b2} = \sqrt{(872072)^2 + (3073524)^2}$$

$$= 3194848.88 \text{ N-mm}$$

Therefore, $M_b = M_{b\max} = M_{b1} = 4209097.26 \text{ N-mm}$

$$D = \left[\frac{16}{\pi \times 40} \left\{ (1.5 \times 4209097.26)^2 + (1.25 \times 1910000)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$= 95.08mm$$

Adopt shaft diameter $D = 100mm(Standard\ size)$

Example 5.17: A commercial steel shaft is required to sustain a torque of 450 Nm and bending moment of 300 Nm. Determine

a) The diameter of the solid shaft required.

b) The dimensions of the hollow shaft taking inside diameter 0.8 times outside diameter.

c) Percentage saving in weight of material by using hollow shaft.

d) Angular deflection.

Solution:

a) For a solid shaft subjected to combined bending and torsion, the diameter D is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ (K_b M_b)^2 + (K_t M_t)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where $\tau_{ed} = 40\, N/mm^2$, *for commercial steel as per ASME code.*

$K_b = 1.5$

$K_t = 1$ ... T(14-2)

$M_b = 300 \text{ N-m} = 300 \times 10^3 \text{ N-mm}$

$M_t = 450 \text{ N-m} = 450 \times 10^3 \text{ N-mm}$

Therefore, $$D = \left[\frac{16}{\pi \times 40} \left\{ (1.5 \times 300 \times 10^3)^2 + (1.0 \times 450 \times 10^3)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Adopt shaft diameter D = 43.27mm

b) For a hollow shaft subjected to combined bending and torsional moments, the outside diameter is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed} \left(1 - k^4\right)} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \quad . \; . \; .$$

E(14-37)

Where $k = \frac{D_i}{D_0} = 0.8$

Therefore, $D_0 = \left[\frac{16}{\pi \times 40\left(1 - 0.8^4\right)} \left\{ \left(1.5 \times 300 \times 10^3\right)^2 + \left(1 \times 450 \times 10^3\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$

$= 51.58mm$

$D_i = 0.8 D_0$

$= 41.27mm$

c) To find the % saving in weight of material

$$\frac{W_H}{W_s} = \frac{\frac{Weight\ of\ hollow\ shaft}{Unit\ length}}{\frac{Weight\ of\ solid\ shaft}{Unit\ length}}$$

$$= \frac{\frac{\pi}{4}\left(d_0^2 - d_i^2\right)\rho}{\frac{\pi}{4}\left(d^2\right)\rho}$$

$$= \frac{d_0^2 - d_i^2}{d^2}$$

$$= \frac{\left(51.58\right)^2 - \left(41.27\right)^2}{\left(43.27\right)^2} = 0.5113$$

Therefore, $W_H = 0.5113 W_s$

Therefore % saving in weight of material

$= (1 - 0.5113)100$

$= 48.87\%$

d) To find θ, the angular deflection of shaft

i) For a solid shaft, the shaft diameter D is given by

$$D = \left[\frac{584 M_t L}{G\theta_s}\right]^{\frac{1}{4}} \qquad \text{... E(14-17)}$$

$$D^4 = \frac{584 M_t L}{G\theta_s}$$

Therefore, $$\theta_s = \frac{584 M_t L}{G D^4}$$

$$= \frac{584 \times 450 \times 10^3 \times 1000}{80 \times 10^3 \times (43.27)^4} = 0.937^0$$

Therefore, Angular deflection for solid shaft $\theta_s = 0.937^0$

ii) For a hollow shaft, the outside diameter D_0 is given by

$$D_0 = \left[\frac{584 M_t L}{G\left(1 - k^4\right)\theta_s}\right]^{\frac{1}{4}} \qquad \text{... E(14-42)}$$

Therefore, $$\theta_h = \frac{584 M_t L}{G D_0^4 \left(1 - k^4\right)}$$

$$= \frac{584 \times 450 \times 10^3 \times 1000}{80 \times 10^3 \times (51.58)^4 \left(1 - 0.8^4\right)} = 0.7861^0$$

Therefore, Angular deflection of hollow shaft $\theta_h = 0.7861^0$

Example 5.18: A steel shaft of rectangular cross section and of size 80mm $\times$ 60mm is subjected to a torsional moment of 400 NM. Determine

i) The maximum shear stress.

ii) The angle of twist in a length of 1000mm. (VTU-Sept/Oct 2000)

Solution:

i) The shear stress induced in the shaft is given by

$$\tau = \frac{M_t}{Z_t}$$

Where $M_t = 400\text{ N-m}\left(given\right)$

$$= 400 \times 10^3 \text{ N-mm}$$

$$Z_t = \frac{I}{C}$$

For rectangular section,

$$I = \frac{bd^3}{12} \quad and \quad C = \frac{d}{2}$$

Therefore, $Z_t = \frac{bd^2}{6}$

$$= \frac{60 \times 80^2}{6} = 384000\, mm^3$$

Therefore, $\tau_{\max} = \frac{400 \times 10^3}{384000} = 10.4\, N/mm^2$

ii) To find the angle of twist θ

$$\frac{M_t}{J} = \frac{G\theta}{l}$$

Therefore, $\theta = \frac{M_t l}{JG}$

$$= \frac{M_t}{G} \times \frac{l}{J}$$

For rectangular section, from table (2-1)

$$\frac{l}{j} = \frac{m\left(b^2 + h^2\right)}{\pi b^3 h^3}$$

For $\frac{h}{b} = 1; \quad m = 3.56$

$\frac{h}{b} = 2; \quad m = 3.5$

When $\frac{h}{b} = 4; \quad m = 3.3$

For the given dimension, $\frac{b}{h} = \frac{60}{80} = \frac{2}{3}$, *referring figure* $m \approx 3.7$

Therefore, $\frac{l}{J} = \frac{3.7\left(60^2 + 80^2\right)}{\pi\left(60\right)^3\left(80\right)^3} = 1.065 \times 10^{-7}$

Therefore, $\theta = \frac{400 \times 10^3}{0.85 \times 10^5} \times 1.065 \times 10^{-7}$

$= \left(5.0117 \times 10^{-7}\right)^0$

Example 5.19: A solid shaft is subjected to a torsional moment of 1500 N-mm. Find the necessary diameter of shaft for an allowable shear stress of 60MPa. An allowable twist of 1^0 for a length of 20 times the shaft diameter. Take $G = 0.8 \times 10^5 \ N/mm^2$. (VTU-July/August 2004)

Solution:

The diameter of solid shaft subjected to torsion is given by

$$D = \left[\frac{16}{\pi\tau_{ed}}\left(K_t M_t\right)\right]^{\frac{1}{3}} \quad \text{... E(14-9)}$$

Where $\tau_{ed} = 60MPa$

$K_t = 1$

$M_t = 1500$ N-m $= 1500 \times 10^3$ N-mm

Therefore, $D = \left[\frac{16}{\pi \times 60} \times 1 \times 1500 \times 10^3\right]^{\frac{1}{3}}$

$= 50.31mm = 51mm$

Based on torsional rigidity,

$$D = \left[\frac{584 M_t l}{G\theta}\right]^{\frac{1}{4}} \qquad \text{... E(14-17)}$$

Where $l = 20D$

$= 20 \times 51 = 1020 mm$

$\theta = 1^0 \, (given)$

$G = 0.8 \times 10^5 \, MPa$

Therefore, $$D = \left[\frac{584 \times \left(1500 \times 10^3\right) \times 1020}{\left(0.8 \times 10^5\right) \times 1}\right]$$

$= 57.81 mm \approx 58 mm$

Adopt $D = 60 mm$ (Large of the two values of D rounded off to standard size)

Example 5.20: Compare the strength of a hollow shaft with that of a solid shaft of the same diameter and material. The diameter ration of hollow shaft is 0.75.

(VTU-July/ August 2004, Dec 08 / Jan 09)

Solution:

It is required to calculate the ratio of $M_t)_{solid\ shaft}$ to that of $M_t)_{hollow\ shaft}$

a) The torque transmitted by a solid shaft of diameter *'d'* is given by

$$M_t)_{solid\ shaft} = \frac{\pi d^3}{16}\tau_s \qquad \text{... (i)}$$

b) The torque transmitted by a hollow shaft having outer diameter $d_0 = d$ and inside diameter d_i is given by

$$M_t)_{hollow\ shaft} = \frac{\pi}{16}\left[\frac{d^4 - d_i^4}{d}\right]\tau_h$$

From equation (i) and (ii),

$$\frac{M_t)_{solid\ shaft}}{M_t)_{hollow\ shaft}} = \frac{\frac{\pi d^3}{16}\tau_s}{\frac{\pi}{16}\left[\frac{d^4 - d_i^4}{d}\right]\tau_h}$$

Since shafts are of same material, $\tau_s = \tau_h$

Therefore, $$\frac{\left.M_t\right)_{solid\ shaft}}{\left.M_t\right)_{hollow\ shaft}} = \frac{d^3}{1} \times \frac{d}{d^4 - d_i^4}$$

$$= \frac{d^4}{d^4 - d_i^4}$$

Or $$\frac{\left.M_t\right)_{solid\ shaft}}{\left.M_t\right)_{hollow\ shaft}} = \frac{d^4 - d_i^4}{d^4}$$

$$= 1 - \left(\frac{d_i}{d}\right)^4$$

$$= 1 - (0.75)^4 = 0.6836$$

Note: $k = \frac{d_i}{d_0} = 0.75$

$$\left.M_t\right)_{hollow\ shaft} = 0.6836\left[\left.M_t\right)_{solid\ shaft}\right]$$

Example 5.21: In an axial flow rotary compressor, the hollow shaft is subjected to a maximum torque of 1500 N-m and a maximum bending moment of 3000 N-m. The inner diameter is 0.4 times outer diameter. The shear stress for the material of shaft is limited to 50MPa. Determine the dimensions of hollow shaft. What will be the shear stress if an axial load of 10kN is taken into account.

Solution:

i) To determine the dimensions of hollow shaft

For a hollow shaft subjected to combined bending and torsion, the outside diameter D_0 is given by

$$D_0 = \left[\frac{16}{\pi\tau_{ed}\left(1-k^4\right)}\left\{\left(K_bM_b\right)^2 + \left(K_tM_t\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad . \; . \; .$$

E(14-37)

Where $\tau_{ed} = 50MPa$

$$k = \frac{D_i}{D_0} = 0.4$$

$$M_b = 3000 \text{ N-m} = 3000 \times 10^3 \text{ N-mm}$$

$$M_t = 1500 \text{ N-m} = 1500 \times 10^3 \text{ N-mm}$$

Assuming steady load,

$$K_b = 1.5, \quad K_t = 1$$

Substituting the values in E(14-37) we get,

$$D_0 = \left[\frac{16}{\pi \times 50\left(1 - 0.4^4\right)} \left\{ \left(1.5 \times 3000 \times 10^3\right)^2 + \left(1 \times 1500 \times 10^3\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

i.e., $D_0 = 80mm$

$$D_i = 0.4 D_0$$

$$= 0.4 \times 80 = 32mm$$

ii) To find the shear stress induced taking axial load into account

The outside diameter of hollow shaft subjected to combined bending, torsion and axial loads is given by

$$D_0 = \left[\frac{16}{\pi \tau_{ed}\left(1 - k^4\right)} \left\{ K_b M_b + \frac{\alpha F D_0}{8}\left(1 + K^2\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

Where $D_0 = 80mm$

$$K = 0.4$$

$$\alpha = 1 \ \textit{for axial tensile load}$$

$$F = 10kN = 10 \times 10^3 N = 10^4 N$$

On substitution,

$$80 = \left[\frac{16}{\pi \tau_{ed}\left(1 - 0.4^4\right)} \left\{ 1.5 \times \left(30 \times 10^5\right) + \frac{1 \times 10^4 \times 80}{8}\left(1 + 0.4^2\right)^2 + \left(1 \times 1500 \times 10^3\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$ i.e.,

$\tau_{ed} = 49.42 \ N/mm^2$

i.e., The shear stress induced in shaft taking axial load into account $= 49.42 \, N/mm^2$

Example 5.22: A solid shaft of diameter 100mm has the same area as that of a hollow shaft of 20mm diameter and of same material. Compare

i) The power transmitted by solid shaft that of hollow shaft assuming same speed.

ii) The angle of twist of hollow shaft with that of solid shaft assuming the same length and shear stress.

Solution:

From data, $Area\ of\ solid\ shaft = Area\ of\ hollow\ shaft$

If D, D_0 and D_i are the diameters of solid shaft and inside and outside diameters of hollow shaft respectively, then

$$D = 100mm, \quad D_0 = 200mm$$

Now equating areas of cross section

$$\frac{\pi}{4} D^2 = \frac{\pi}{4}\left(D_0^2 - D_i^2\right)$$

i.e., $D^2 = D_0^2 - D_i^2$

$$100^2 = 200^2 - D_i^2$$

Therefore, Inside diameter of hollow shaft

$$D_i = 173.21mm$$

i) The power transmitted by shaft is directly proportional to torque transmitted

Therefore, $$\frac{M_t)_s}{M_t)_h} = \frac{\frac{\pi D^3}{16}\tau_s}{\frac{\pi}{16}\frac{\left(D_0^4 - D_i^4\right)}{D_0}\tau_h}; \qquad \tau_s = \tau_h$$

$$= \frac{D^3 D_0}{D_0^4 - D_i^4}$$

$$= \frac{100^3 \times 200}{200^4 - (173.21)^4} = 0.2858$$

Therefore, $\left.M_t\right)_s = 0.2858 \left.M_t\right)_h$

i.e., the power transmitted by solid shaft is 28.58% of the power transmitted by hollow shaft for the given dimensions.

ii) To find ratio of angle of twist of hollow shaft with that of solid shaft

We have, $\frac{M_t}{J} = \frac{G\theta}{l}$

Therefore, $\theta = \frac{M_t l}{JG}$

$$\theta_s = \frac{M_{ts} l_s}{J_s G_s}; \qquad \theta_h = \frac{M_{th} l_h}{J_h G_h};$$

$$\frac{\theta_h}{\theta_s} = \frac{\frac{M_{th} l_h}{J_h G_h}}{\frac{M_{ts} l_s}{J_s G_s}}$$

$G_s = G_h$ and $l_h = l_s$

Therefore, $\frac{\theta_h}{\theta_s} = \frac{M_{th}}{J_h} \times \frac{J_s}{M_{ts}}$

$$= \frac{M_{th}}{M_{ts}} \times \frac{J_s}{J_h}$$

$$= \frac{\frac{\pi\left(D_0^4 - D_i^4\right)}{16 D_0}\tau_h}{\frac{\pi\left(D^3\right)}{16}\tau_s} \times \frac{\frac{\pi D^4}{32}}{\frac{\pi\left(D_0^4 - D_i^4\right)}{32}}$$

$$= \frac{\dfrac{\left(D_0^4 - D_i^4\right)}{D_0}}{\dfrac{D^3}{1}} \times \frac{\dfrac{D^4}{1}}{\dfrac{\left(D_0^4 - D_i^4\right)}{1}}$$

$$= \frac{\left(D_0^4 - D_i^4\right)}{D_0} \times \frac{1}{D^3} \times \frac{D^4}{1} \times \frac{1}{\left(D_0^4 - D_i^4\right)}$$

$$\frac{\theta_h}{\theta_s} = \frac{D^4}{D_0 D^3}$$

$$= \frac{D}{D_0} = \frac{100}{200} = 0.5$$

Therefore, $\dfrac{\theta_h}{\theta_s} = \dfrac{1}{2}$ *i.e.,* $\theta_s = 2\theta_h$

Example 5.23: A commercial steel shaft 1m long supported between bearings carries a pulley of diameter 600mm weighing 800N keyed to the shaft 300mm to the right of left bearing and is driven by a motor placed directly below it with a maximum belt tension of 3000N. Another pulley of diameter 800mm weighing 1000N is mounted 300mm to the left of right bearing and delivers power to another pulley mounted on a parallel shaft in front of it by means of a flat belt drive. The angle of lap of both pulleys is 180^0 and the coefficient of friction between pulleys and belt as 0.3. Determine the shaft diameter assuming steady load condition.

Solution:

The shaft is subjected to combined bending and torsion and hence the shaft diameter is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where $\tau_{ed} = 40MPa$, *for commercial steel according to ASME code*

$K_b = 1.5$

$K_t = 1$

To find M_t, the torque

Consider the pulley

Ratio of belt tensions $= \dfrac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 180 \times \frac{\pi}{180}} = 2.57$

Here $T_1 = T_{\max} = 3000N$

Therefore, $T_2 = \dfrac{T_1}{e^{\mu\theta}} = \dfrac{3000}{2.57} = 1167.32N$

Therefore torque, $M_t = (T_1 - T_2)\, r_{Pulley1}$

$$= (3000 - 1167.32)\frac{600}{2}$$

$$= 549804 \text{ N-mm}$$

To find M_b, the maximum bending moment on shaft

Consider pulley 1

Horizontal load on pulley 1 = 0.

Vertical load on pulley 1 $= (T_1 + T_2 + W_p) \downarrow$

$$= (3000 + 1167.32 + 800) \downarrow$$

$$= 4967.32N \downarrow$$

Consider pulley 2

Horizontal load on pulley 2 $= (T_1' + T_2') \rightarrow$

To find T_1' *and* T_2'

Ratio of belt tensions $\dfrac{T_1'}{T_2'} = 2.57$

Note: Torque transmitted by both pulleys is the same.

Therefore, $\left.M_t\right)_{Pulley1} = \left.M_t\right)_{Pulley2}$

$$549809 = (T_1' - T_2')\, r_{Pulley2}$$

$$= (2.57T_2' - T_2')\frac{800}{2}$$

$T_2' = 875.5N$

$T_1' = 2626.48N$

Horizontal load on pulley 2 $= 3501.98N \rightarrow$

Vertical load on pulley 2 $= Wp_2 \downarrow = 1000N \downarrow$

Vertical load diagram

$$R_{AV} + R_{BV} = 4967.32 + 1000$$

$$= 5967.32N \uparrow$$

Taking moments about A, and equating sum of clockwise moments to sum of anticlockwise moments,

$$R_{BV} \times 1000 = 4967.32 \times 300 + 1000 \times 700$$

Therefore, $R_{BV} = 2190.2N$

$R_{AV} = 3777.12N$

Vertical BMD

Bending moment $M_b = 0$ at bearing A and B

M_b at pulley 1 $= 3777.12 \times 300$ N-mm $= 1133136$ N-mm

M_b at pulley 2 $= 2190 \times 300$ N-mm $= 657060$ N-mm

Horizontal load diagram

$$R_{AH} + R_{BH} = 3501.98N$$

Taking moments about A,

$$R_{BH} \times 1000 = 3501.989N$$

Therefore, $R_{BH} = 2451.39N$

$R_{AH} = 1050.59N$

$M_t)_{at\ Pulley1} = M_{b1} = 1176152.1$ N-mm

$M_t)_{at\ Pulley2} = M_{b2} = 985187.6$ N-mm

Therefore, $M_b = M_{b1} = 1176152.1$ N-mm (*The maximum value*)

Therefore, $$D=\left[\frac{16}{\pi\times 40}\left\{(1.5\times 1176152.1)^2+(1\times 549804)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$=61.74mm$$

Adopt $D=65mm$

Example 5.24: A steel shaft 1meter long supported between bearings carries two cast iron pulleys keyed to it. Pulley 1, 600mm in diameter weighing 800N is mounted 300mm to the right of left bearing and receives 25kW at 1400 rpm from another parallel shaft behind it and parallel it by means of a flat belt drive. Power from the shaft is transmitted thorough a second pulley of diameter 800mm weighing 1000N in front of it by a horizontal flat belt. The second pulley is mounted 400mm to the left of right bearing. Taking $\sigma_u=500MPa,\ \sigma_y=310MPa,$ calculate the diameter of the shaft required using ASME code. Assume the load to be steady. Use ration of belt tensions = 3:1.

Solution:

The shaft is subjected to combined bending and torsion and hence the diameter of shaft is given by

$$D=\left[\frac{16}{\pi\tau_{ed}}\left\{(K_bM_b)^2+(K_tM_t)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

a) To find τ_{ed}, the permissible shear stress

The material of the shaft has

$$\sigma_u=500MPa$$

$$\sigma_y=310MPa$$

Now, 18% of $\sigma_u=0.18\times 500=90MPa$

30% of $\sigma_y=0.3\times 310=93MPa$

$\tau_{ed}=75\%$ of the least of the above two values as per ASME code when key is present.

Therefore, $\tau_{ed}=0.75\times 90=67.5MPa$

$K_b=1.5,\quad K_t=1(\textit{for steady load})$

b) To find M_t, the torque transmitted

$$M_t = \frac{9550N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 25 \times 10^3}{1400} \text{ N-mm}$$

$$= 170535.7 \text{ N-mm}$$

c) To find M_b, the maximum bending moment on shaft

i) Consider pulley 1

Horizontal load on pulley $= (T_1 + T_2) \leftarrow$

Where, Ratio of belt tensions $\frac{T_1}{T_2} = \frac{3}{1}$

i.e., $T_1 = 3T_2$ ... (i)

Also $M_t = (T_1 - T_2) r_1$

Where $r_1 = Radius\ of\ pulley\ 1$

i.e., $170535.7 = (3T_2 - T_2)\frac{600}{2}$ ... (ii)

$$T_2 = 284.23N$$

$$T_1 = 852.69N$$

Therefore, Horizontal load on Pulley 1

$$= (852.69 + 284.23) \leftarrow$$

$$= 1136.92N \leftarrow$$

Vertical load on Pulley 1

$$= W_{p1} \downarrow = 800N \downarrow$$

ii) Consider pulley 2

Horizontal load on pulley $= (T_1' + T_2') \rightarrow$

Where $\frac{T_1'}{T_2'} = 3$

i.e., $T_1' = 3T_2'$ ... (iii)

Torque $M_t = (T_1' - T_2')r_2$

$$= (3T_2' - T_2')\frac{800}{2} = 170535.7 \quad \text{... (iv)}$$

From equation (iii) and (iv),

$$T_2' = 213.17N$$

$$T_1' = 3T_2' = 639.51N$$

Therefore, *Horizontal load on pulley 2*

$$= 639.51 + 213.17 = 852.68N \rightarrow$$

Vertical load on pulley 2

$$= W_{p2} \downarrow = 1000N \downarrow$$

d) To draw horizontal load and bending moment diagrams

Equating vertical loads,

$$R_A + R_B + 1136.92 = 852.68N$$

Therefore, $R_{AH} + R_{BH} = -284.24N$

Taking moments about *A*,

$$R_{BH} \times 1000 + 1136.92 \times 300 = 852.68 \times 600$$

Therefore, $R_{BH} = 170.532N$

$$R_{AH} = -454.772N$$

To draw vertical load diagram (Load vertical bending moment diagram)

$$R_{AV} + R_{BV} = 1000 + 810 = 1800$$

Taking moments about *A*,

$$R_{BV} \times 1000 = 1000 \times 600 + 800 \times 300$$

Therefore, $R_{BV} = 840N$

$$R_{AV} = 960N$$

Resultant bending moment M_{b1} at pulley 1 $= 318674.32$ N-mm

Resultant bending moment M_{b2} at pulley 2 $= 342854.18$ N-mm

Therefore, $M_b = M_{b2} = 342854.18$ N-mm $\left(\textit{Higher of the two values of } M_b\right)$

Therefore,
$$D = \left[\frac{16}{\pi \times 67.5}\left\{\left(1.5\times 342854.18\right)^2 + \left(1\times 170535.7\right)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$= 34.448mm$$

Adopt $D = 35mm$

Example 5.25: A hollow steel shaft 800mm long with outside diameter 80mm and inside diameter 45mm held rigidly at one end has a 600mm diameter wheel attached to the other end. Determine

i) The bending stress induced in shaft when a tangential force of 600N is applied to the wheel rim.

ii) The torsional shear stress in the shaft due to 600N force.

Solution

To find bending stress (σ_b)

i) Bending moment M_b due to 600N is

$$M_b = Fl$$

$$= 600 \times 800 = 48 \times 10^5 \text{ N-mm}$$

Bending stress $\sigma_b = \dfrac{M_b}{Z_b}$

Where $Z_b = \textit{Section modulus} = \dfrac{I}{c}$

For hollow shaft

$$Z_b = \frac{\frac{\pi}{64}\left(d_0^4 - d_i^4\right)}{\frac{d_0}{2}}$$

$$= \frac{\pi}{32}\left(\frac{d_0^4 - d_i^4}{d_0}\right)$$

$$= \frac{\pi}{32}\frac{\left(80^4 - 45^4\right)}{80}$$

$$= 45233.26 mm^3$$

Therefore, $\sigma_b = \frac{M_b}{Z_b}$

$$= \frac{48\times10^4}{45233.26} = 10.61\ N/mm^2$$

ii) To find torsional shear stress (τ)

Torsional shear stress $\tau = \frac{M_t}{Z_t}$

Where $M_t = F_t r$

$$= 600\times\frac{600}{2} = 18\times10^4 \text{ N-mm}$$

$$Z_t = \frac{J}{c}$$

$$= \frac{\pi}{16}\frac{\left(d_0^4 - d_i^4\right)}{d_0}$$

$$= \frac{\pi}{16}\frac{\left(80^4 - 45^4\right)}{80}$$

$$= 90466.52 mm^3$$

Therefore, $\tau = \frac{18\times10^4}{90466.52} = 1.99\ N/mm^2$

Example 5.26: A steel shaft (with $\sigma_y = 309.9 MPa$) 1.5m long between bearings carries a 1kN pulley at is midpoint. The pulley is keyed to the shaft and receives 20kW power at 200

rpm. The belt drive is horizontal and ratio of belt tensions is 3:1. The diameter of pulley is 600mm. Calculate the necessary shaft diameter. Take $K_b = 1.75, K_t = 1.25$ and factor of safety = 2.
(VTU-July/August 2004)

Solution:
The shaft is subjected to combined bending and torsional loads

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $K_b = 1.75, \; K_t = 1.25 (given)$

Torque $M_t = \dfrac{9550 N \times 10^3}{n}$ N-mm

$$= \frac{9550 \times 20 \times 10^3}{200} = 9550 \times 10^2 \text{ N-mm}$$

To calculate M_b, the maximum bending moment on shaft

Horizontal load on pulley $= (T_1 + T_2) \rightarrow$

Where $\dfrac{T_1}{T_2} = 3$

i.e., $T_1 = 3T_2$ ... (i)

Also Torque $M_t = (T_1 - T_2) r_{Pulley}$ ... (ii)

$9550 \times 10^2 = (3T_2 - T_2)\dfrac{600}{2} \rightarrow$ ... (iii)

From equation (i) and (ii)

$T_2 = 1591.67N$

$T_1 = 4775N$

Therefore, Horizontal load on pulley $= 6366.67N \rightarrow$

Vertical load on pulley $W_p = 1kN \downarrow = 1000N \downarrow$

To draw BM diagrams

Because of symmetry,

$$R_{AH} = R_{BH} = \frac{6366.67}{2} = 3183.33N$$

At pulley, $M_{b1} = R_{AH} \times 750$

$$= 3183.33 \times 750 \text{ N-mm}$$

Again because of symmetry,

$$R_{AV} = R_{BV} = \frac{1000}{2} = 500N$$

At pulley, $M_{b2} = R_{AV} \times 750$

$$= 50 \times 750 \text{ N-mm}$$

Maximum bending moment

$$M_b = \sqrt{(3183.33 \times 750)^2 + (500 \times 750)^2}$$

$$= 2416768.4 \text{ N-mm}$$

To find τ_{ed}, the design shear stress

$$\sigma_y = 309.9MPa$$

$$\tau_y = \frac{\sigma_y}{2}$$

$$= \frac{309.9}{2} = 154.95MPa$$

$$\tau_{ed} = \frac{\tau_y}{factor\ of\ safety}$$

$$= \frac{154.95}{2} = 77.48MPa$$

$$D = \left[\frac{16}{\pi \times 77.48} \left\{ (1.75 \times 2416768)^2 + (1.5 \times 955000)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$D = 66.1mm$$

Adopt $D = 70mm$

Example 5.27: A steel shaft 1 metre long supported between bearings carries a CI pulley of diameter 400mm weighing 800N keyed in between (at centre) bearings and receives 25kW at 1400 rpm from another pulley mounted behind it by means of a flat belt drive inclined at 60^0 to the horizontal sloping upwards. Power from the shaft is transmitted to another pulley of diameter 800mm weighing 1000N overhanging the right bearing by 200mm to another pulley mounted on a parallel shaft infront of it. Taking the ration of belt tensions as 3:1, determine the diameter of shaft required. The permissible shear stress is not to exceed 60MPa.

Solution:
The shaft is subjected to combined bending and torsion. The diameter of shaft is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where, $K_b = 1.5$

$K_t = 1$ *(Assuming steady load)*

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 N \times 10^3}{1400} = 170535.71 \text{ N-mm}$$

$$\tau_{ed} = 60 MPa \left(given \right)$$

a) To find M_b, the maximum bending moment

Consider pulley 1 mounted between bearings

Horizontal load on pulley $= \left(T_1' + T_2' \right) \cos 60 \leftarrow$

Vertical load on pulley $= \left(T_1' + T_2' \right) \text{c} \sin 60 \sim W_{p1}$

$= 1476.87 \uparrow \sim 800 \downarrow = 676.87 N \uparrow$

b) To find T_1' *and* T_2'

Ratio of belt tensions $\frac{T_1'}{T_2'} = 3$

Therefore, $T_1' = 3T_2'$ ... (i)

Also, torque $M_t = (T_1' - T_2')r_1$

$$170535.71 = (3T_2' - T_2')\frac{400}{2} = 400T_2' \quad \text{... (ii)}$$

From equation (i) and (ii),

$$T_2' = 426.34N$$

$$T_1' = 3T_2' = 1279N$$

Therefore, Horizontal load on pulley 1 $= (T_1' + T_2') = 852.68N \leftarrow$

Vertical load on pulley 1 $= 676.87N \uparrow$

c) Consider pulley 2

Horizontal load on pulley $= (T_1 + T_2) \rightarrow$

Vertical load on pulley $= W_{p2} \downarrow = 1000N \downarrow$

d) To find T_1 and T_2

We have $\frac{T_1}{T_2} = 3$

i.e., $T_1 = 3T_2$ ... (iii)

Also $M_t = (T_1 - T_2)r_2$

Torque $= (3T_2 - T_2)\frac{800}{2}$

$= 800T_2 = 170535.72$ N-mm ... (iv)

From equation (iii) and (iv)

$$T_2 = 213.17N$$

$$T_1 = 639.51N$$

Therefore, Horizontal load on pulley 2 $= (T_1 + T_2) = 852.68N \rightarrow$

Vertical load on pulley 2 $= 1000N \downarrow$

e) To draw load and bending moment diagrams

i) To draw horizontal load diagram and horizontal bending moment diagram

From HLD, $R_{AH} = R_{BH} + 852.68 = 852.68$

Therefore, $R_{AH} = R_{BH} = 0 \;\; is \;\; R_{AH} = -R_{BH}$

Taking moments about *A*,

$$852.68 \times 1200 = 852.68 \times 500 + R_B \times 1000$$

Therefore, $R_{BH} = 596.88NN$

$$R_{AH} = -R_{BH} = -596.88N$$

ii) Vertical load diagram and vertical bending moment diagram

$$R_{AV} + R_{BV} + 676.87 = 1000$$

Therefore, $R_{AV} + R_{BV} = 323.13N$

Taking moments about *A*,

$$1000 \times 1200 = 676.87 \times 500 + R_{BV} \times 1000$$

Therefore, $R_{BV} = 861.57N$

$$R_{AV} = -584.44N$$

Resultant BM at pulley 1

$$= \sqrt{(596.88 \times 500)^2 + (584.44 \times 500)^2}$$

$$= 417682.85 \text{ N-mm}$$

Resultant BM at bearing *B*

$$M_{b2} = \sqrt{(852.68 \times 200)^2 + (1000 \times 200)^2}$$

$$= 262835.55 \text{ N-mm}$$

Therefore, $M_b = M_{b1} = 417682.82 \text{ N-mm} \left(\textit{i.e., maximum bending moment} \right)$

Therefore, $$D = \left[\frac{16}{\pi \times 60} \left\{ (1.5 \times 417682.85)^2 + (1 \times 170535.71)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}}$$

$$D = \sqrt[3]{55143.86}$$

$$D = 38.06mm$$

Adopt $D = 40mm$

Example 5.28: A commercial steel shaft 1 meter long supported between bearings carries two spur gears keyed to shaft. Gear A of module 5mm having 40 teeth with 14 ½^0 pressure angle is keyed to shaft 100mm to the right of left end bearing and receives 10kW at 1400 rpm from another gear mounted on a parallel shaft behind it. Power from the shaft is transmitted through gear B of module 5mm and 60 teeth keyed to the shaft 200mm to the left of right bearing to another pinion mounted on a parallel shaft infront of it. Assuming the loads are suddenly applied with minor shock, determine the standards size of shaft required based on maximum shear stress theory.

Solution:

The shaft is subjected to combined bending and torsional loads and hence the diameter of solid shaft based on maximum shear stress theory is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b\right)^2 + \left(K_t M_t\right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where $D = Shaft\ diameter$

$\tau_{ed} = 40MPa\left(for\ commercial\ shaft\ with\ keyway\ as\ per\ ASME\ code \right)$

$$K_b = 1.75$$

$$K_t = 1.5$$

$$M_t = \frac{9550N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 10 \times 10^3}{1440} = 66319.44 \text{ N-mm}$$

a) To find M_b the maximum bending moment on shaft

i) Consider gear A,

Assuming the gear *A* rotating in the clockwise direction when viewed from left,

Vertical load on Gear A

$$F_{t1} = \frac{M_t}{r_1} \downarrow$$

$$= \frac{66319.44}{100} = 663.19N \downarrow$$

Horizontal load on Gear A

$$F_{r1} = F_{t1} \tan \beta$$
$$= 663.19 \times \tan 14\tfrac{1}{2}$$
$$= 171.51N \rightarrow$$

ii) Consider gear B

The vertical load on Gear B

$$F_{t2} \uparrow = \frac{M_t}{r_2}$$
$$= \frac{66319.44}{150} N \uparrow$$
$$= 442.13N \uparrow$$

The horizontal load on Gear B

$$F_{r2} = F_{t2} \tan \beta \leftarrow$$
$$= 442.13 \times \tan 14\tfrac{1}{2} \leftarrow$$
$$= 114.34N \leftarrow$$

b) To draw load and bending moment diagrams

i) HLD and HBMD

$$R_{AH} + R_{BH} + F_{r2} = F_{r1}$$
$$R_{AH} + R_{BH} + 114.34 = 171.51$$

Therefore, $R_{AH} + R_{BH} = 57.17N$

Taking moments about A,

$$R_{BH} \times 1000 + F_{r2} \times 800 = 171.51 \times 100$$
$$R_{BH} \times 1000 + 114.34 \times 800 = 171.51 \times 100$$

Therefore, $R_{BH} = -74.32N$

$$R_{AH} = 131.49N$$

Bending moment at Gear A $= 131.49 \times 100$ N-mm

Bending moment at Gear B $= 74.32 \times 200$ N-mm

ii) Vertical load diagram and vertical bending moment diagram

$$R_{AV}+R_{BV}+442.13=663.19N$$

Therefore, $R_{AV}+R_{BV}=221.06N$

Taking moments about A,

$$R_{BV}\times1000+442.130\times800=663.19\times100$$

Therefore, $R_{BV}=-287.39N$

$$R_{AV}=221.06-R_{BV}$$

$$=221.06+287.39=+508.45N$$

Resultant bending moment M_{b1} at Gear A

$$=\sqrt{(131.49\times100)^2+(508.45\times100)^2}$$

$$=52517.71 \text{ N-mm}$$

Resultant bending moment M_{b2} at Gear B

$$=\sqrt{(74.32\times200)^2+(287.39\times200)^2}$$

$$=59368.84 \text{ N-mm}$$

Therefore, $M_b=M_{b\max}=M_{b2}=59368.84$ N-mm

Therefore, $D=\left[\frac{16}{\pi\times40}\left\{(1.75\times59368.84)^2+(1.5\times66319.44)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$

$$=26.36mm$$

Adopt $D=30mm$

Example 5.29: A steel shaft 0.8m long supported between bearings carries two pulleys. Pulley 1 is 600mm in diameter weighs 1kN keyed to the shaft 100mm to the right of left bearing and receives 25kW at 1400 rpm from another pulley mounted on a parallel shaft behind it at an angle of 30^0 to the vertical directed below the horizontal. Power from the shaft is transmitted through a second pulley of diameter 800mm weighing 1.5kN mounted 200mm to the left of right bearing to another shaft infront of it by another flat belt inclined downward at 45^0 to the vertical. Taking the ratio of belt tensions as 3:1, determine the diameter of shaft required based on maximum shear stress theory. Restrict the shear stress in shaft to 50MPa.

Solution:

The shaft is subjected to combined bending and torsion and hence the diameter of solid shaft is given by

$$D = \left[\frac{16}{\pi \tau_{ed}} \left\{ \left(K_b M_b \right)^2 + \left(K_t M_t \right)^2 \right\}^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad \text{... E(14-12)}$$

Where $\tau_{ed} = 50 MPa$

$K_b = 1.5$

$K_t = 1$

$$M_t = \frac{9550 N \times 10^3}{n} \text{ N-mm}$$

$$= \frac{9550 \times 25 \times 10^3}{1400} = 170535.71 \text{ N-mm}$$

a) To find $M_{b,}$ the maximum bending moment

i) Consider pulley 1

Horizontal load $= \left(T_1 + T_2 \right) \sin 30 \leftarrow$

Vertical load $= \left(T_1 + T_2 \right) \cos 30 + W_{p1} \downarrow$

Where, Ratio of belt tensions $\frac{T_1}{T_2} = 3$

Therefore $T_1 = 3T_2$

Also torque $M_t = \left(3T_2 - T_1 \right) r_1$

$$170535.71 = 2T_2 \times \frac{600}{2}$$

Therefore, $T_2 = 284.23N$

$T_1 = 852.69N$

Therefore, *Horizontal load* $= 567.45N \leftarrow$

Vertical load $= \left(852.69 + 284.23 \right) \cos 30 + 1000 \downarrow$

$= 1984.6 \downarrow$

ii) Consider pulley 2

Horizontal load $= \left(T_1' + T_2' \right) \sin 45 \rightarrow$

Vertical load $= \left(T_1' + T_2'\right)\cos 45 + W_{p2} \downarrow$

To find T_1' *and* T_2'

$$\frac{T_1'}{T_2'} = 3$$

Therefore. $T_1' = 3T_2'$ ... (i)

Also $M_t = \left(T_1' - T_2'\right) r_2$

$$170535.71 = \left(3T_2' - T_2'\right)\frac{800}{2} \quad \text{... (ii)}$$

From equation (i) and (ii)

$$T_2' = 213.17N$$

$$T_1' = 639.51N$$

Therefore, *Horizontal load* $= \left(639.51 + 213.71\right)\sin 45 = 602.94N \rightarrow$

Vertical load $= \left(639.51 + 213.71\right)\cos 45 + 1500 = 2102.94N \downarrow$

Horizontal load and horizontal bending moment diagrams

$$R_{AH} + R_{BH} + 568.45 = 602.94 \quad \text{... (iii)}$$

Therefore, $R_{AH} + R_{BH} = 34.49N$

Taking moments about A,

$$R_{BH} \times 800 + 568.45 \times 100 = 602.94 \times 600 \quad \text{... (iv)}$$

From equation (iii) and (iv)

$$R_{BH} = 381.15N$$

Therefore, $R_{AH} = -346.66N$

c) Vertical load diagram and vertical bending moment diagram

$$R_{AV} + R_{BV} = 1984.6 + 210.94$$

$$= 4087.54N \quad \text{... (v)}$$

Taking moments about A,

$$R_{BV} \times 800 = 1984.6 \times 100 + 2102.94 \times 600 \quad \text{... (vi)}$$

From equation (v) and (iv)

$$R_{BV} = 1825.28N$$

$$R_{AV} = 2262.26N$$

d) To calculate M_b, the maximum bending moment on shaft

Resultant bending moment at Pulley 1

$$M_{b1} = \sqrt{(346.66\times100)^2 + (2262.26\times100)^2}$$

$$= 228866.63mm$$

Resultant bending moment at Pulley 2

$$M_{b2} = \sqrt{(381.15\times200)^2 + (1825.28\times200)^2}$$

$$= 372930.15 \text{ N-mm}$$

Therefore, $M_b = M_{b1} = 372930.15$ N-mm *(Maximum value of bending moment)*

Therefore, $$D = \left[\frac{16}{\pi\times50}\left\{(1.5\times372930.15)^2 + (1\times170535.71)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$$= 39.05mm$$

Adopt $D = 40mm$

Example 5.30: A mild steel shaft 0.8m long supported between bearings carries two pulleys keyed to it and overhang the bearing by 200mm. Pulley 1 is 500mm in diameter weighs 1000N and receivers 25kW at 1400 rpm from an equal size pulley mounted on a parallel shaft infront of it by means of a belt drive inclined at 30^0 to the horizontal sloping upwards, when seen from left. Power from the shaft is transmitted through the second pulley of diameter 750mm weighing 1.5kN to another equal size pulley mounted on a parallel shaft behind it by means of another flat belt parallel to the first pulley but sloping downwards. Taking coefficient of friction between belts and pulleys as 0.3, using maximum shear stress theory, determine the standard size of shaft required by limiting the permissible shear stress to 60MPa.

Solution: